6.851 Advanced Data Structures (Spring'12)

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Problem 3   *Due: Thursday, Mar. 8*

Be sure to read the instructions on the assignments section of the class web page.

Geometric basics. Recall that in class we introduced the geometric view for binary search tree execution. Using only this view, prove that for any set of m queries on n items, there is a BST that will answer the queries in total time $O(m \lg n)$. You should reason only about point sets, not about BSTs.

Working-set is harder. In class we introduced the *entropy bound* and the *working-set property* for BSTs. The entropy bound holds if all searches in the BST have amortized time $O(\sum_{k=1}^{n} p_k \lg \frac{1}{p_k})$, where p_k is the fraction of the time that key k is queried. The working-set property holds if the time to search for an element x_i is $O(\lg t_i)$, where t_i is the number of elements queried since the last access to x_i. Prove that any BST with the working-set property also has the entropy bound.