

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

*Mathematics for Computer Science*  
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# The Well Ordering Principle, II

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Albert R Meyer February 13, 2012 Lec 2M.1

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## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

*Proof:* (by contradiction) Suppose {nonproducts} is nonempty. By WOP, there is a **least**  $m > 1$  that is a nonproduct. This  $m$  is not prime (else is a product of 1 prime)

Albert R Meyer February 13, 2012 Lec 2M.2

6	9	13	7
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## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

...So  $m = j \cdot k$  for integers  $j, k$  where  $m > j, k > 1$ . Now  $j, k < m$  so both are prime products:  
 $j = p_1 \cdot p_2 \cdots p_{94}$      $k = q_1 \cdot q_2 \cdots q_{213}$

Albert R Meyer February 13, 2012 Lec 2M.3

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## Prime Products

*Thm:* Every integer  $> 1$  is a product of primes.

...now  
 $m = j \cdot k = p_1 \cdot p_2 \cdots p_{94} \cdot q_1 \cdot q_2 \cdots q_{213}$   
 is prime product, **contradiction**.  
 So {counterexamples} =  $\emptyset$ . **QED**

Albert R Meyer February 13, 2012 Lec 2M.4

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## Well Ordered Postage

available stamps: 5¢ 3¢

$n$  is **postal** if can make  $(n+8)$ ¢ postage from 3¢ & 5¢ stamps.

Albert R Meyer February 13, 2012 Lec 2M.5

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## Well Ordered Postage



available stamps: 5¢ 3¢

*Thm:* Every number is **postal**.  
 Prove by WOP. Suppose **not**.  
 Let  $m$  be **least** counterexample.

Albert R Meyer February 13, 2012 Lec 2M.6

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### Well Ordered Postage

available stamps:  

That is,



- $m$  is **not** postal,
- any number  $< m$  is postal

$5\text{¢}$ 
 $3\text{¢}$


 Lec 2M.7

6	9	13	7
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### Well Ordered Postage


0 is postal:  


so  $m \neq 0$

 Lec 2M.8


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### Well Ordered Postage

$m \neq 1$ : 

$m \neq 2$ : 



Hence,  $m \geq 3$ .

 Lec 2M.9


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### Well Ordered Postage

Now  $m-3$  is a number  $< m$ , so is postal. But then  $m$  is postal too:


+  =  $m+8\text{¢}$

$(m-3)+8\text{¢}$  **contradiction!**

 Lec 2M.10