Mathematics for Computer Science

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The Well Ordering Principle, II

Prime Products

Thm: Every integer > 1 is a product of primes.

Proof: (by contradiction) Suppose {nonproducts} is nonempty. By WOP, there is a least $m > 1$ that is a nonproduct. This m is not prime (else is a product of 1 prime)

Prime Products

Thm: Every integer > 1 is a product of primes.

…So $m = j \cdot k$ for integers j,k where $m > j,k > 1$. Now $j,k < m$ so both are prime products:

$j = p_1 \cdot p_2 \cdots p_{94}$ $\quad k = q_1 \cdot q_2 \cdots q_{213}$

Prime Products

Thm: Every integer > 1 is a product of primes.

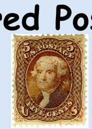
…now

$m = j \cdot k = p_1 \cdot p_2 \cdots p_{94} \cdot q_1 \cdot q_2 \cdots q_{213}$

is prime product, contradiction.

So {counterexamples} = $\emptyset$. QED

Well Ordered Postage

available stamps:  

5¢ 3¢

n is postal if can make $(n+8)$¢ postage from 3¢ & 5¢ stamps.

Well Ordered Postage

available stamps:  

5¢ 3¢

Thm: Every number is postal.

Prove by WOP. Suppose not. Let m be least counterexample.

Well Ordered Postage

available stamps:

5¢ 3¢

That is,

- m is not postal,
- any number $< m$ is postal

Well Ordered Postage

0 is postal:

so $m \neq 0$

Well Ordered Postage

$m \neq 1$:

$m \neq 2$:

Hence, $m \geq 3$.

Well Ordered Postage

Now m-3 is a number $< m$, so is postal. But then m is postal too:

+ 3¢ $= m$+8¢

(m-3)+8¢ contradiction!