

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Trees



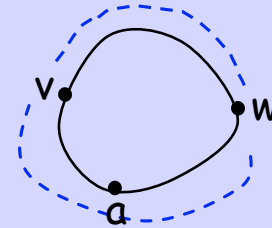
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tree-def.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cycles in simple graphs

A **cycle** is a closed walk of **length** > 2 that doesn't cross itself:



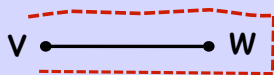
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tree-def.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cycles in simple graphs

length > 2 implies that going back & forth over an edge is **not** a cycle



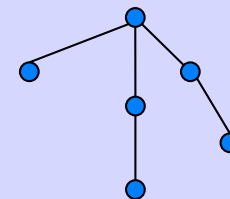
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tree-def.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Trees

A **tree** is a connected graph with **no cycles**.

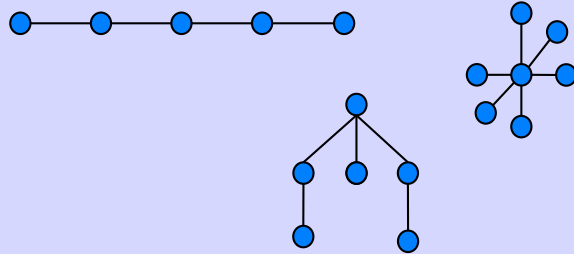


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tree-def.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

More Trees



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tree-def.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Come up all the time

- family trees
- search trees
- game trees
- parse trees
- spanning trees



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tree-def.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Lots of kinds

- rooted
- ordered
- binary
- complete
- directed



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tree-def.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Focus on "pure" trees

unordered,
unrooted,
undirected



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tree-def.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges

An edge is a **cut edge** if removing it from the graph disconnects two vertices.

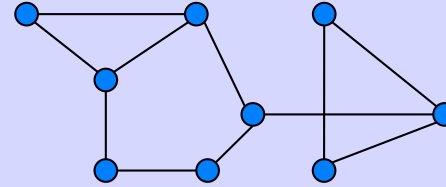


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tree-def.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges

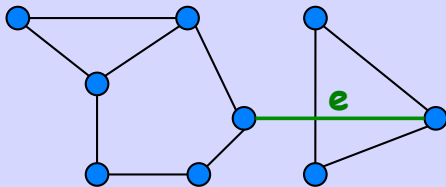


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tree-def.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges



e is a cut edge

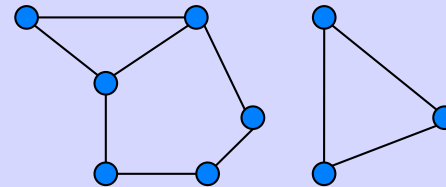


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tree-def.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges



deleting **e** gives two components

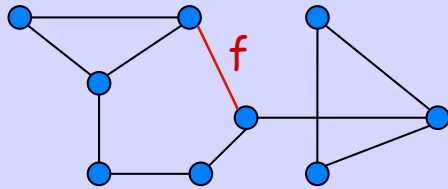


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tree-def.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges



f is **not** a cut edge

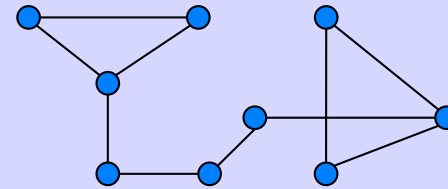


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tree-def.14

6	9	13	7
12	10	5	
3	1	4	14
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Cut Edges



still connected with
edge f deleted



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tree-def.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges

So a connected graph is
2-edge connected iff
it has **no cut edge**.



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tree-def.16

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Cut Edges and Cycles

Lemma: An edge is a
not a cut edge iff
it is **on a cycle**.



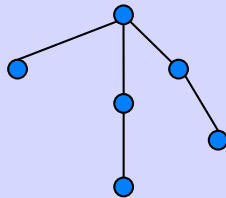
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tree-def.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

alternative tree definition

A **tree** is a connected graph with **every edge a cut edge**.



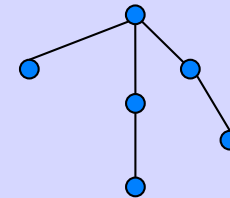
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tree-def.18

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

alternative tree definition

A **tree** is a connected graph that is **edge-minimal**.



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tree-def.19

6	9	13	7
12		10	5
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more equivalent definitions

- connected graph with n vertices and $n-1$ edges
- an **edge-maximal** acyclic graph
- graph with a **unique** path between **any 2** vertices



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tree-def.22