

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

• Mathematics for Computer Science
MIT 6.042J/18.062J

Stable Matching: The Mating Ritual (day by day)



Albert R Meyer. April 3, 2013

stable-ritual.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mating Ritual

Morning: boy serenades favorite girl



Billy Bob



Brad



Angelina



Albert R Meyer. April 3, 2013

stable-ritual.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mating Ritual

Morning: boy serenades favorite girl
Afternoon: girl **rejects** all but favorite

If you're not Brad
take a hike!



Billy Bob



Brad



Angelina



Albert R Meyer. April 3, 2013

stable-ritual.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mating Ritual

- Morning: boy serenades **his** favorite girl
- Afternoon: girl rejects all but **her** favorite boy
- Evening: rejected boy writes off girl



Billy Bob



Albert R Meyer. April 3, 2013

stable-ritual.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual

Stop when no girl rejects.
Each girl marries her favorite suitor (if any).



Albert R Meyer. April 3, 2013

stable-ritual.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Stable Marriages

Termination:

There exists a Wedding Day.

Partial Correctness:

Everyone is married.

Marriages are stable.



Albert R Meyer. April 3, 2013

stable-ritual.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Termination**

total # names remaining
on boys' lists:

strictly decreasing

& \mathbb{N} -valued

So \exists Wedding Day



Albert R Meyer. April 3, 2013

stable-ritual.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Correctness**



Albert R Meyer. April 3, 2013

stable-ritual.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: girls improve

Lemma:

A girl's favorite tomorrow will be at least as desirable to her as today's.
 ...because today's favorite will stay until she rejects him for someone better.



Albert R Meyer. April 3, 2013

stable-ritual.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: girls improve

Lemma:

The rank of a girl's favorite is weakly increasing



Albert R Meyer. April 3, 2013

stable-ritual.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: boys get worse

Lemma:

A boy's favorite tomorrow will be no more desirable to him than today's.
 ...because boys work straight down their lists.



Albert R Meyer. April 3, 2013

stable-ritual.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: boys get worse

Lemma:

The rank of girl a boy serenades is weakly decreasing



Albert R Meyer. April 3, 2013

stable-ritual.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **invariant**

If G is not on B 's list, then she has a better current favorite.

Proof: When G rejected B she had a better suitor (her favorite that day), and her favorites never get worse.



Albert R Meyer. April 3, 2013

stable-ritual.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

On Wedding Day

Each girl has ≤ 1 suitor.
(by def of wedding day)
Each boy is married, or has no girls on his list.



Albert R Meyer. April 3, 2013

stable-ritual.15

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **No bigamy**

No husband sharing ...because boys serenade
one girl at a time.



Albert R Meyer. April 3, 2013

stable-ritual.16

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Everyone marries**

Everyone is married on wedding day

Proof: By **contradiction**.

If B is not married, his list is empty.

By **invariant**, all girls have favorites better than B —so they do have a favorite: all **girls** are married. Since no bigamy, all **boys** are married.



Albert R Meyer. April 3, 2013

stable-ritual.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob won't be in rogue couple with
case 1: a girl G **on** his final list,
 since he's already married to the
 best of them.



Albert R Meyer. April 3, 2013

stable-ritual.18

6	5	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob won't be in rogue couple with
case 2: a girl G **not on** his list,
 since by invariant, G likes her
 spouse better than Bob.



Albert R Meyer. April 3, 2013

stable-ritual.19

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual: **Stable marriages**

Marriages are Stable:

Bob is not in a rogue couple
 There are **no rogue couples**



Albert R Meyer. April 3, 2013

stable-ritual.20