

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

• *Mathematics for Computer Science*
MIT 6.042J/18.062J

Optimal Stable Matching



Albert R. Meyer, 2015

September 29, 2015

optimal.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual

Who does better,
boys or girls?

Girls' suitors get better,
and boys' sweethearts
get worse



Albert R. Meyer, 2015

September 29, 2015

optimal.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mating Ritual

Who does better,
boys or girls?

Girls' suitors get better,
and boys' sweethearts
get worse, so girls do
better? **No!**



Albert R. Meyer, 2015

September 29, 2015

optimal.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Feasible spouses

Nicole and Keith are
feasible spouses when
they are married in
some set of stable
marriages.



Albert R. Meyer, 2015

September 29, 2015

optimal.5

6	9	13	7
12		10	5
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15	8	11	2

Optimal spouses

Nicole is **optimal** for
Keith when she is
Keith's **highest ranked**
feasible spouse



Albert R. Meyer, 2015

September 29, 2015

optimal.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Boys **win** the Ritual

In Mating Ritual:
Every boy marries his
optimal wife.



Albert R. Meyer, 2015

September 29, 2015

optimal.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Boys **win** the Ritual

Follows from **Invariant**
No girl has rejected
a feasible spouse



Albert R. Meyer, 2015

September 29, 2015

optimal.8

6	9	13	7
12		10	5
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15	8	11	2

Boys **win** the Ritual

Assume the **Invariant**.
Boy's wife is top-listed girl
on Wedding Day.



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September 29, 2015

optimal.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Boys **win** the Ritual

Assume the **Invariant**.

Boy's wife is top-listed girl on Wedding Day. Deleted girls not feasible, so wife is highest ranked feasible.

optimal wife



Albert R. Meyer, 2015

September 29, 2015

optimal.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Verify the **Invariant**

No girl has rejected a feasible spouse



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September 29, 2015

optimal.11

6	9	13	7
12		10	5
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Verify the **Invariant**

Suppose Invariant holds when Nicole rejects Bob. Must show that Bob is not feasible for Nicole.



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optimal.12

6	9	13	7
12		10	5
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15	8	11	2

Verify the **Invariant**

Nicole rejects Bob ...because Tom is serenading her and she prefers Tom to Bob.



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optimal.14

6	9	13	7
12		10	5
3	1	4	14
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Verify the Invariant

By Invariant, Tom's feasible wives still on his list.



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optimal.15

6	9	13	7
12		10	5
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Verify the Invariant

By Invariant, Tom's feasible wives still on his list. Nicole is top of list, so ranks above all Tom's other feasibles.



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optimal.16

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Verify the Invariant

So Nicole cannot stably marry Bob, because



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optimal.18

6	9	13	7
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Verify the Invariant

So Nicole cannot stably marry Bob, because she and Tom would be rogue:
She prefers Tom to Bob, and Tom would prefer her to whoever his wife is.



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optimal.19

6	9	13	7
12	10	5	
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Verify the **Invariant**

So rejected suitor Bob
is indeed not feasible:
Invariant is preserved!



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optimal.20

6	9	13	7
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Girls **lose** the Ritual

Easier argument shows
each girl gets worst
possible spouse.



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September 29, 2015

optimal.21

6	9	13	7
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15	8	11	2

Boys **win**, Girls **lose**

In Mating Ritual:

Every boy marries his
optimal wife.

Every girl marries her
pessimal husband.



Albert R. Meyer, 2015

September 29, 2015

optimal.22

6	9	13	7
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Stable Marriage

- More questions, rich theory
- other stable marriages possible? - can be **many**
 - do better by lying?
boys **-No!** girls **-Yes!**



Albert R. Meyer, 2015

September 29, 2015

optimal.23