

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

• Mathematics for Computer Science
MIT 6.042J/18.062J

Stable Matching



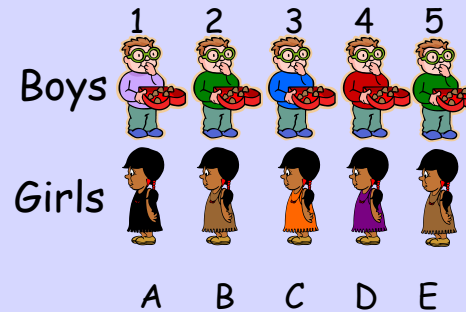
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stable.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

A Marriage Problem



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12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage Preferences

- | Boys | Girls |
|-----------|-----------|
| 1 : CBEAD | A : 35214 |
| 2 : ABECD | B : 52143 |
| 3 : DCBAE | C : 43512 |
| 4 : ACDBE | D : 12345 |
| 5 : ABDEC | E : 23415 |



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stable.3

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12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Preferences

- 1: CBEAD
- 2: ABECD
- 3: DCBAE
- 4: ACDBE
- 5: ABDEC

Try "greedy" strategy for boys



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12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Preferences

- 1: ~~C~~BEAD
- 2: ABED
- 3: D~~B~~AE
- 4: A~~D~~BE
- 5: ABDE~~C~~

Marry Boy 1 with Girl C
(his 1st choice)



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stable.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Preferences

- 2: ABED
- 3: D BAE
- 4: A DBE
- 5: ABDE



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stable.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Preferences

- 2: ABED
- 3: DBAE
- 4: AD BE
- 5: ABDE



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stable.7

6	9	13	7
12	10	5	
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15	8	11	2

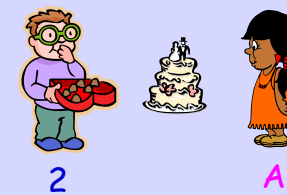
Stable Marriage

Preferences

- 2: ~~A~~BED
- 3: D~~B~~AE
- 4: A~~D~~BE
- 5: ABDE

Next:

Marry Boy 2 with Girl A
(his remaining 1st choice)



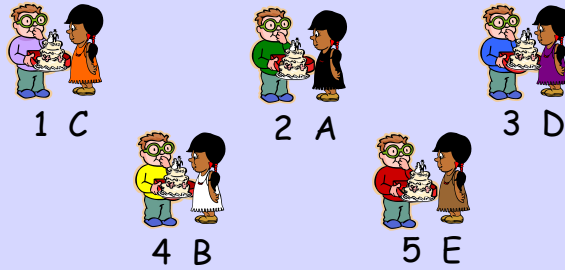
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stable.8

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12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Final "boy greedy" marriages



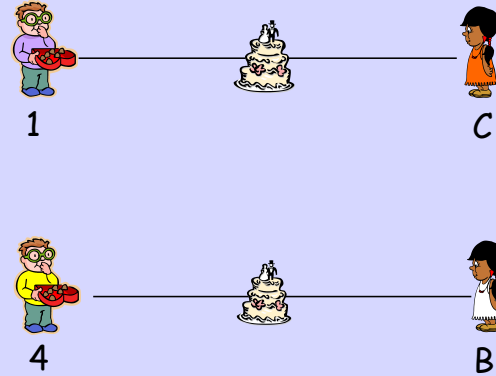
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3	1	4	14
15	8	11	2

Stable Marriage

Trouble!



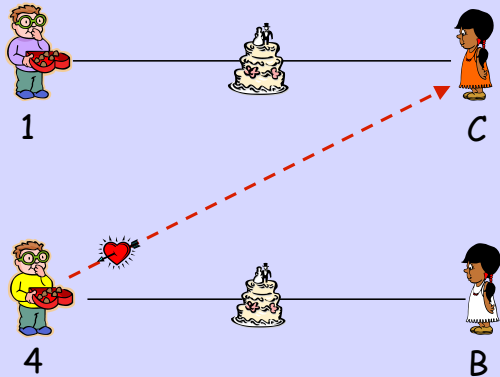
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stable.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Boy 4 likes Girl C better than his wife.



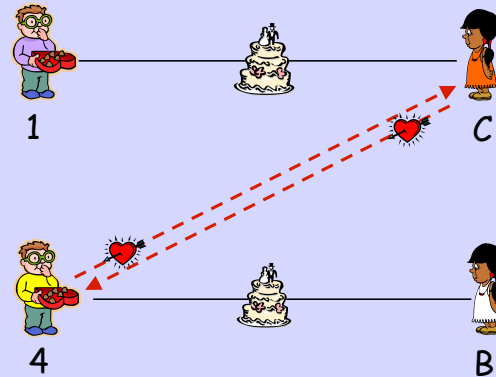
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stable.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

and vice-versa

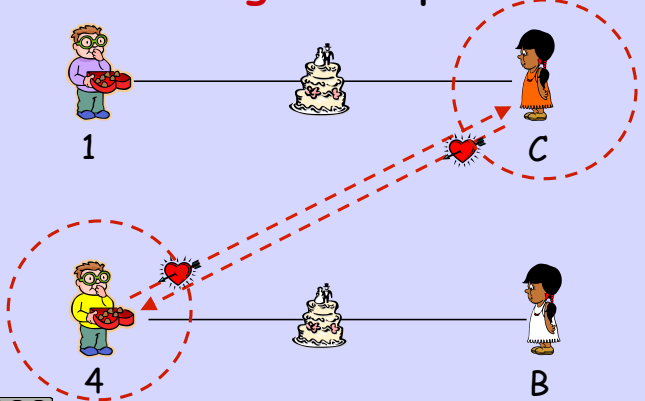


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stable.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage a **rogue** couple



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stable.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Stable Marriage Problem:

Marry everyone without any rogue couples!

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stable.14

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

Let's try it!

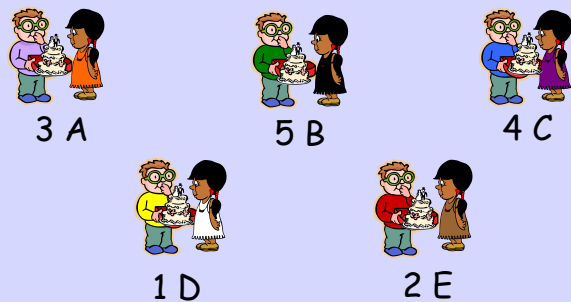
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stable.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage I.



3 A

5 B

4 C

1 D

2 E

all girls get 1st choice

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stable.16

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12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage II



5 A



2 B



4 C



3 D



1 E

"boy optimal"



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stable.17

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Stable Marriage

More than a puzzle:

- College Admissions
(original Gale & Shapley paper, 1962)
- Matching Hospitals & Residents.
- Matching Dance Partners.



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stable.18

6	9	13	7
12	10	5	
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15	8	11	2

Stable Marriage



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stable.19