

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

Proving Valid Formulas



Albert R Meyer, 2017

February 14, 2018

sound.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Proving Validity

Instead of truth tables,
can try to **prove** valid
formulas symbolically using
axioms and deduction rules.



Albert R Meyer, 2017

February 14, 2018

sound.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sound Proof Systems

A system for proving
formulas is **sound**, when
every provable formula
is **valid**.



Albert R Meyer, 2017

February 14, 2018

sound.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Proving Validity

Several sound propositional
proof systems have a few
valid axioms and just **one**
proof rule:

modus ponens



Albert R Meyer, 2017

February 14, 2018

sound.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens rule

antecedents

$P, P \text{ IMPLIES } Q$

Q

conclusion



Albert R Meyer, 2017

February 14, 2018

sound.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sound Rules...

...preserve **validity**:
if all the antecedents are
valid, then conclusion is
valid.



Albert R Meyer, 2017

February 14, 2018

sound.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens is sound

If P is **valid**,
and $P \text{ IMPLIES } Q$ is **valid**,
then Q must be **valid**.



Albert R Meyer, 2017

February 14, 2018

sound.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens is sound

Proof:



Albert R Meyer, 2017

February 14, 2018

sound.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens is sound

If P is T for truth assignment A ,
 and $P \text{ IMPLIES } Q$ is T for A ,
 then Q is T for A .
 (by truth table for IMPLIES)



Albert R Meyer, 2017

February 14, 2018

sound.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

modus ponens is sound

If P is T for truth assignment A ,
 and $P \text{ IMPLIES } Q$ is T for A ,
 then Q is T for A .

But P and $P \text{ IMPLIES } Q$ are T for
 every A , so Q must be T for
 every A -- that is, Q is *valid*.



Albert R Meyer, 2017

February 14, 2018

sound.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Provable Formulas

Start with axioms and
 derive more formulas by
 repeatedly applying the
 proof rules to previously
 derived formulas



Albert R Meyer, 2017

February 14, 2018

sound.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sound Proof Systems

Theorem: If proof *rules*
 are *sound*, and axioms are
valid, then every provable
 formula is *valid*.



Albert R Meyer, 2017

February 14, 2018

sound.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sound Proof Systems

Theorem: If proof rules are **sound**, and axioms are **valid**, then the whole proof system is **sound**



Albert R Meyer, 2017

February 14, 2018

sound.13

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Complete proof systems

A proof system is **complete** when every valid formula is provable. These sound proof systems are indeed also complete.



Albert R Meyer, 2017

February 14, 2018

propositional logic.14

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Deduction & Algebra

Text and MITx unit describe a complete proof system based on **proving equivalences using algebra-style rules**

MITx unit "Propositional Logic" describes **modus ponens based deduction system**



Albert R Meyer, 2017

February 14, 2018

sound.15

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Validity/SAT still difficult!

Known proof systems **in general** are no better than truth tables.
No **efficient method** for verifying **validity** is known.



Albert R Meyer, 2017

February 14, 2018

propositional logic.23