

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Simple Graphs: k-Connectivity



Albert R Meyer, April 5, 2013

k-connect.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Edge Connectedness

Def: vertices  $v, w$  are  
**k-edge connected**  
if they remain connected  
whenever **fewer than k**  
edges are deleted.

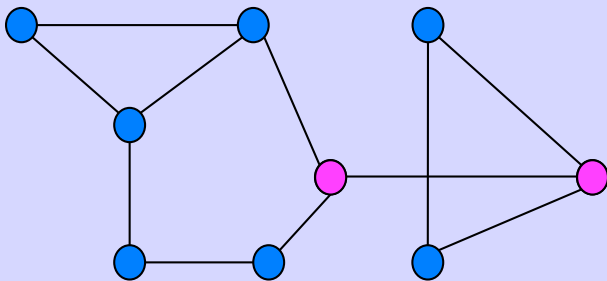


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k-connect.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-edge Connectedness



1-edge connected

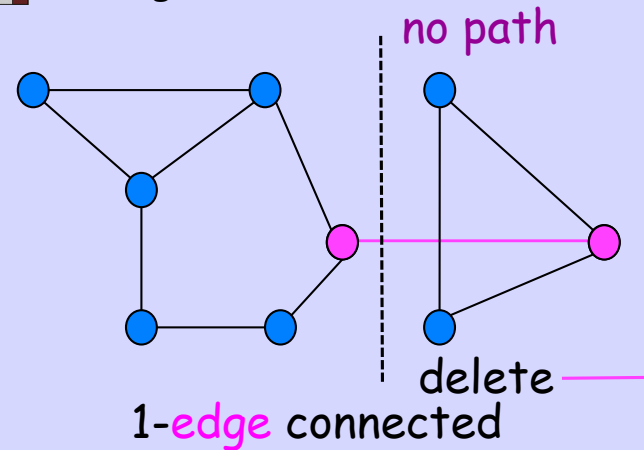


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k-connect.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-edge Connectedness



1-edge connected

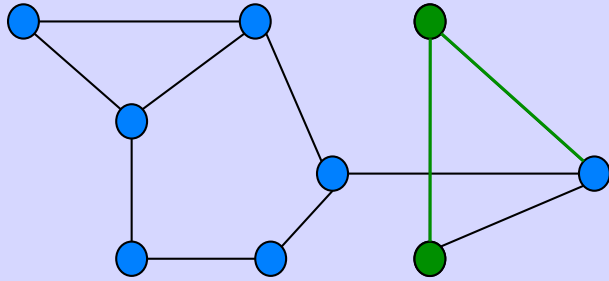


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k-connect.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Edge Connectedness



2-edge connected

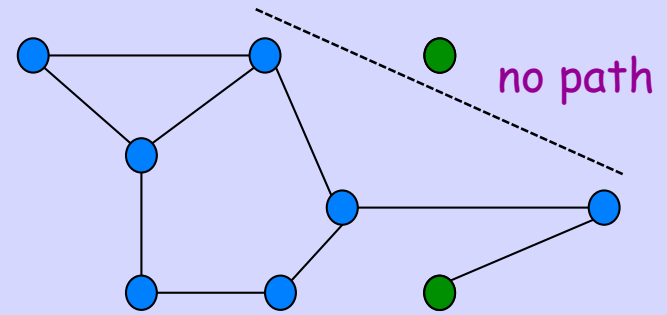


Albert R Meyer, April 5, 2013

k-connect.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Edge Connectedness



2-edge connected

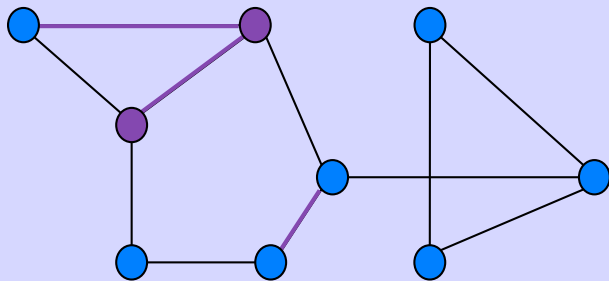


Albert R Meyer, April 5, 2013

k-connect.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Edge Connectedness



3-edge connected

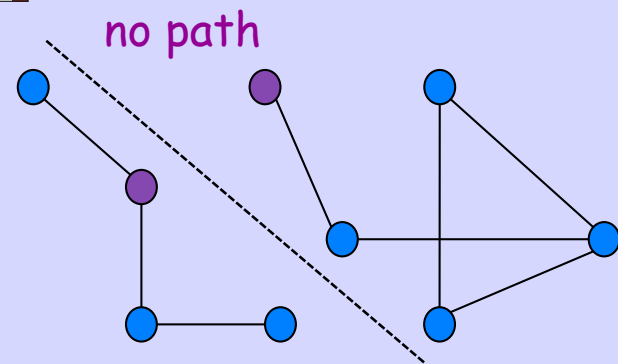


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k-connect.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Edge Connectedness



3-edge connected



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k-connect.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-edge Connectedness

Def: A graph is  
**k-edge connected**  
 iff every two vertices  
 are **k-edge connected**.



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k-connect.9

6	9	13	7
12	10	5	
3	1	4	14
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## Edge Connectedness

Connectivity measures **fault tolerance** of a network:  
 how many connections can  
 fail without cutting off  
 communication?



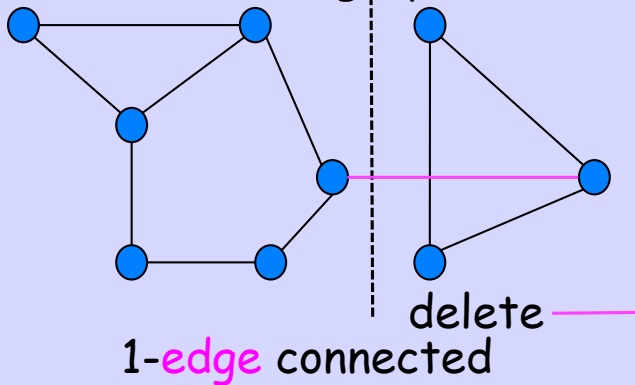
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k-connect.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-edge Connectedness

this whole graph is



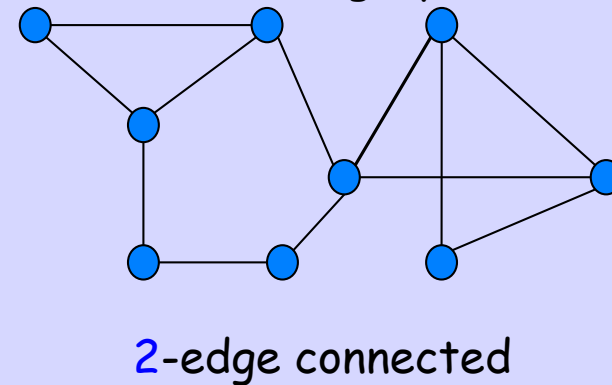
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k-connect.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-edge Connectedness

this whole graph is



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k-connect.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## k-vertex Connectedness

k-vertex  
connectedness  
defined similarly



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k-connect.13

6	9	13	7
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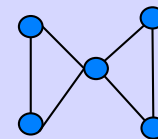
## k-vertex Connectedness

k-vertex connected

IMPLIES

k-edge connected

not conversely:



2-edge connected

1-vertex connected



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k-connect.14

6	9	13	7
12	10	5	
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## k-vertex Connectedness

$K_n$  is the complete graph on  $n$  vertices.

$K_n$  is  $(n-1)$ -vertex connected.



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k-connect.15

6	9	13	7
12	10	5	
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## k-vertex Connectedness

The  $n$ -dimensional hypercube  $H_n$

$V(H_n) ::= \{0,1\}^n$

$\langle u-v \rangle$  an edge IFF  $u, v$  differ in 1 place



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k-connect.16

6	9	13	7
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## $k$ -vertex Connectedness

$H_n$  is  $n$ -vertex  
connected.  
(class problem)



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k-connect.17

6	9	13	7
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## Menger's Theorem

$k$ -connected vertices  
will be connected by  $k$   
non-overlapping paths



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k-connect.18