

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Simple Graphs: Isomorphism



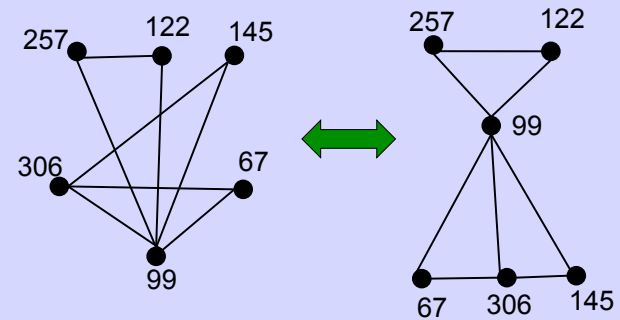
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isomorphism.1

6	9	13	7
12	10	5	
3	1	4	14
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The Graph Abstraction

Same graph (different layouts)



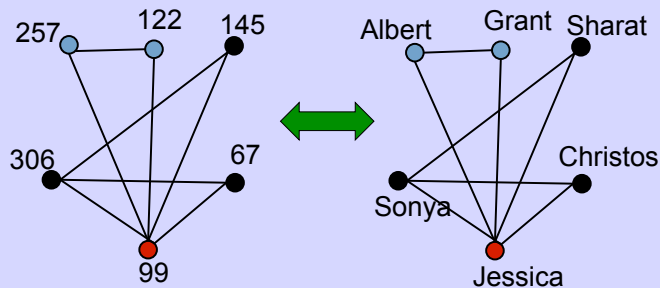
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isomorphism.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

The Graph Abstraction

Same graph (different labels)



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isomorphism.3

6	9	13	7
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The Graph Abstraction

All that matters
are the **connections**:
graphs with the
same connections
are **isomorphic**



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isomorphism.4

6	9	13	7
12	10	5	
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Isomorphism

two graphs are **isomorphic**
when there is an
edge-preserving
matching
of their vertices.

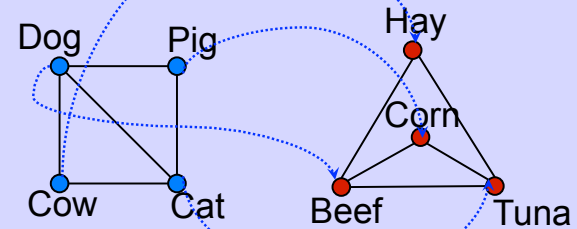


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isomorphism.5

6	9	13	7
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Are these isomorphic?



$f(\text{Dog}) = \text{Beef}$ $f(\text{Cow}) = \text{Hay}$
 $f(\text{Cat}) = \text{Tuna}$ $f(\text{Pig}) = \text{Corn}$

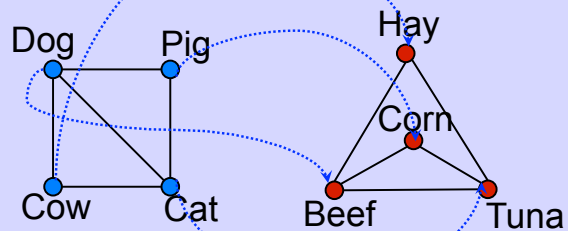


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isomorphism.6

6	9	13	7
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Edges preserved?

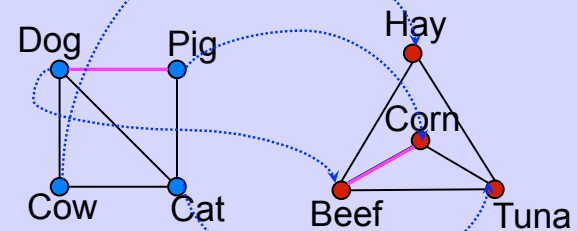


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isomorphism.7

6	9	13	7
12	10	5	
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Edges preserved? **YES!**

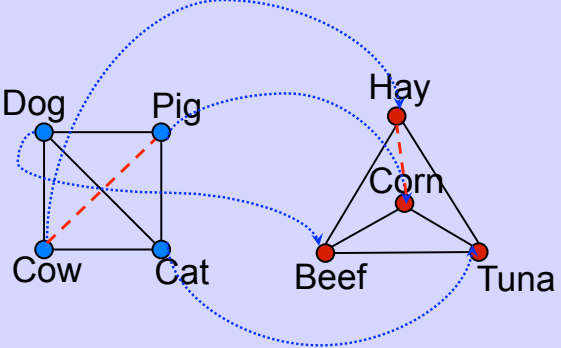


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isomorphism.8

6	9	13	7
12	10	4	
3	1	4	14
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Nonedges preserved? YES!



isomorphic!

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isomorphism.9

6	9	13	7
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Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 means edge-preserving vertex matching:

\exists bijection $f: V_1 \rightarrow V_2$ with
 $u-v$ in E_1 IFF $f(u)-f(v)$ in E_2

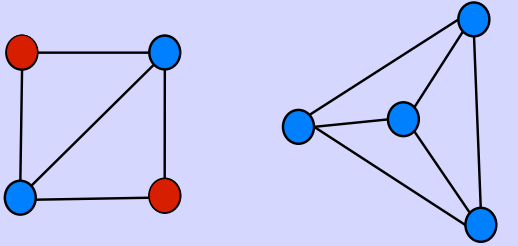
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isomorphism.10

6	9	13	7
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Nonisomorphism



degree 2

all degree 3

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isomorphism.11

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Proving nonisomorphism

If some property preserved by isomorphism differs for two graphs, then they're not isomorphic:

- # of nodes,
- # of edges,
- degree distributions,

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isomorphism.12

6	9	13	7
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Finding an isomorphism?

many possible mappings: **large search**

can use properties **preserved** by isomorphisms as a guide, for example:

- a **deg 4 vertex adjacent to a deg 3** can only match with
- a **deg 4 vertex also adjacent to a deg 3**

but even so...

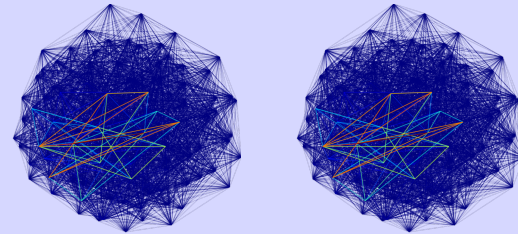


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isomorphism.13

6	9	13	7
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Are these two graphs isomorphic?



...nothing known is *sure* to be much faster than searching thru all bijections for an isomorphism



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isomorphism.14