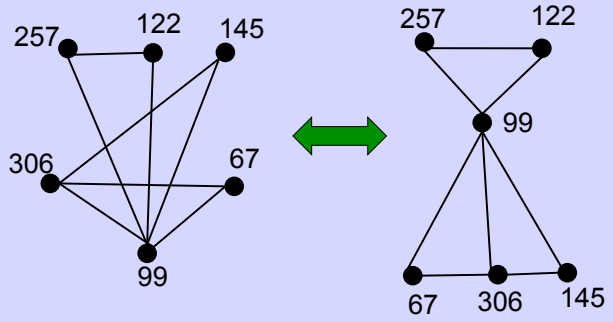
Mathematics for Computer Science

MIT 6.042J/18.062J

Simple Graphs: Isomorphism

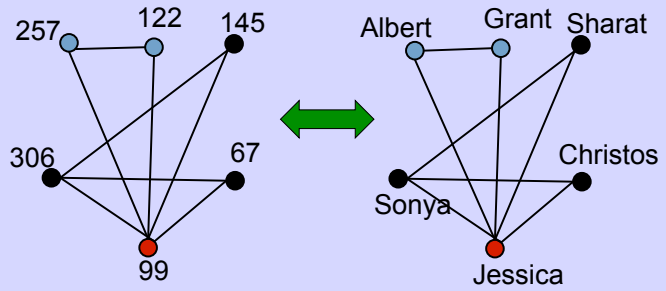
The Graph Abstraction

Same graph (different layouts)



The Graph Abstraction

Same graph (different labels)



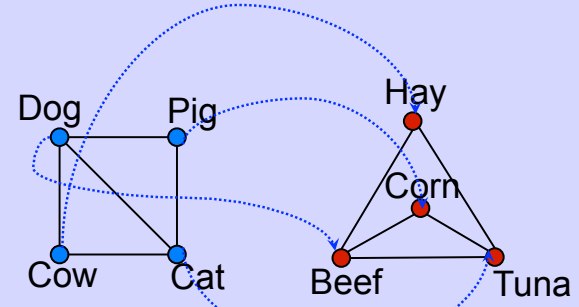
The Graph Abstraction

All that matters are the connections: graphs with the same connections are isomorphic

Isomorphism

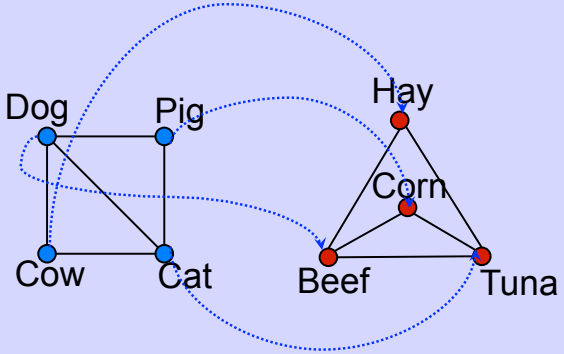
two graphs are isomorphic when there is an edge-preserving matching of their vertices.

Are these isomorphic?

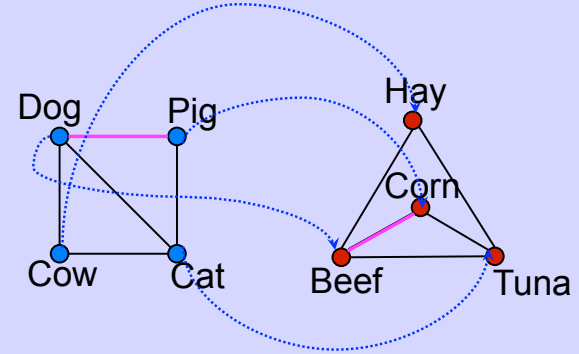


f(Dog) = Beef f(Cow) = Hay
f(Cat) = Tuna f(Pig) = Corn

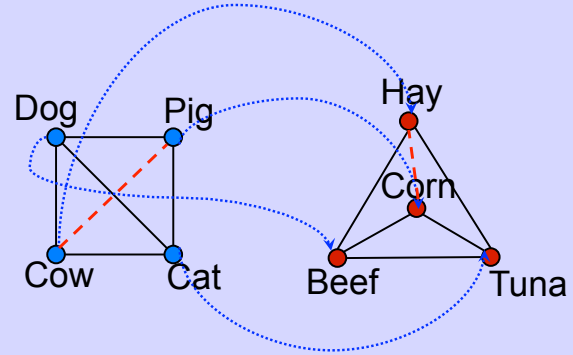
Edges preserved?



Edges preserved? YES!



Nonedges preserved? YES!



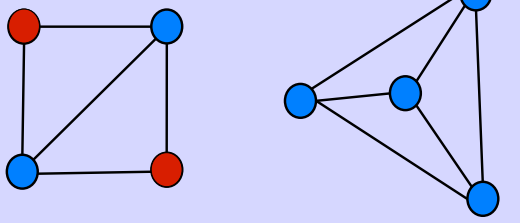
isomorphic!

Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 means edge-preserving vertex matching:

$\exists$ bijection $f: V_1 \to V_2$ with
u—v in E_1 IFF $f(u)$—$f(v)$ in E_2

Nonisomorphism



degree 2 all degree 3

Proving nonisomorphism

If some property preserved by isomorphism differs for two graphs, then they're not isomorphic:

# of nodes,
# of edges,
degree distributions, ...

Finding an isomorphism?

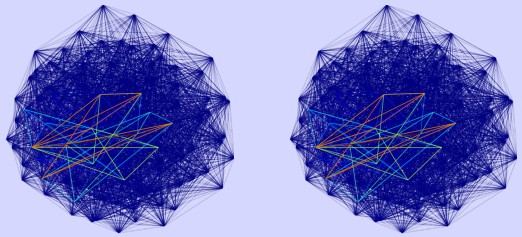
many possible mappings: large search

can use properties preserved by isomorphisms as a guide, for example:

- a deg 4 vertex adjacent to a deg 3
  can only match with
- a deg 4 vertex also adjacent to a deg 3

but even so…

Are these two graphs isomorphic?



…nothing known is *sure* to be much faster than searching thru all bijections for an isomorphism