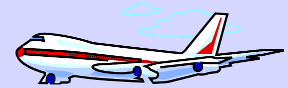
Mathematics for Computer Science
MIT 6.042J/18.062J

Simple Graphs: Coloring

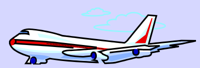
Flight Gates



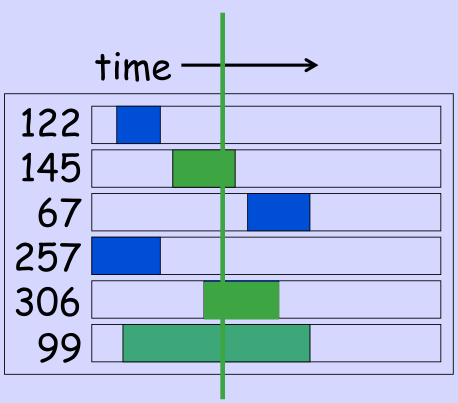
flights need gates, but times overlap.

how many gates needed?

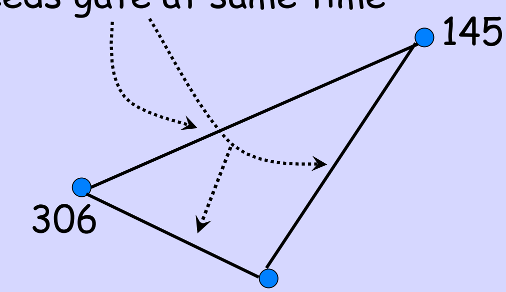
Airline Schedule



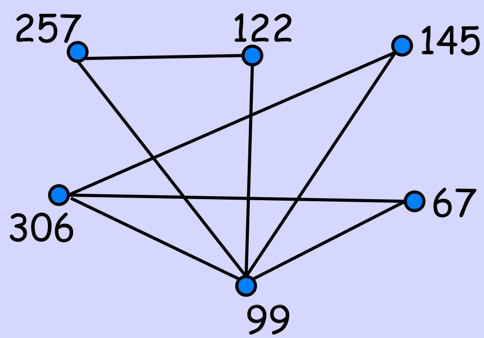
Flights



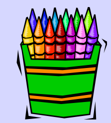
Conflicts Among 3 Flights



Model all Conflicts with a Graph



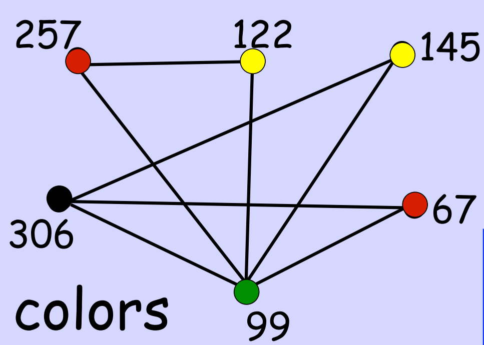
Color the vertices



Color vertices so that adjacent vertices have different colors.

min # distinct colors needed = min # gates needed

Coloring the Vertices

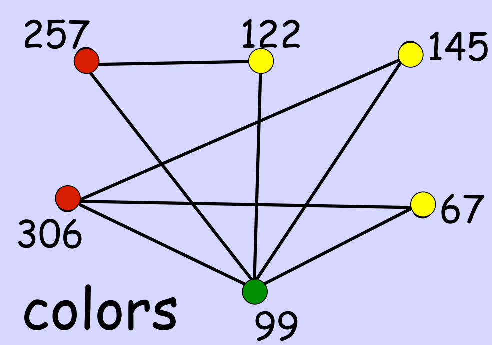


assign gates:

- 🔴 257, 67
- 🟡 122,145
- 🟢 99
- ⚫ 306

4 colors
4 gates

Better coloring



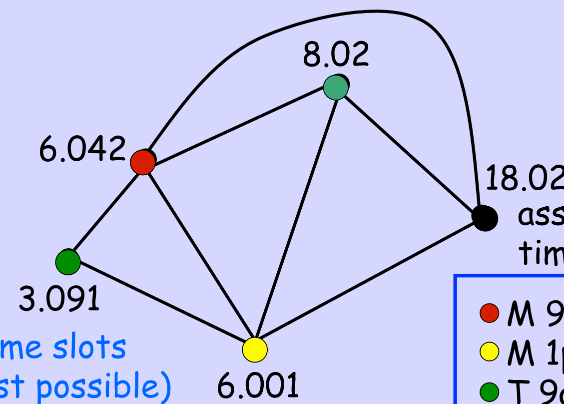
3 colors
3 gates

Final Exams

subjects conflict if student takes both, so need different time slots.

how short an exam period?

Model as a Graph



assign times:

- M 9am
- M 1pm
- T 9am
- T 1pm

4 time slots (best possible)

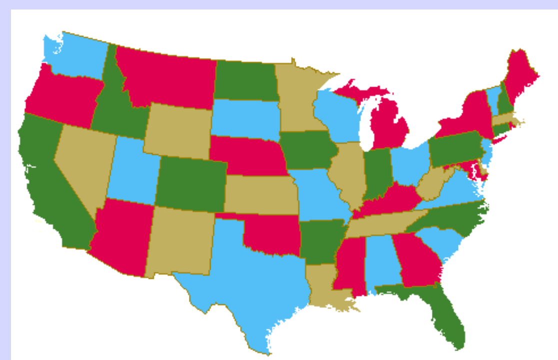
Conflicting Allocation Problems

separate habitats to house different species of animals, some incompatible with others?

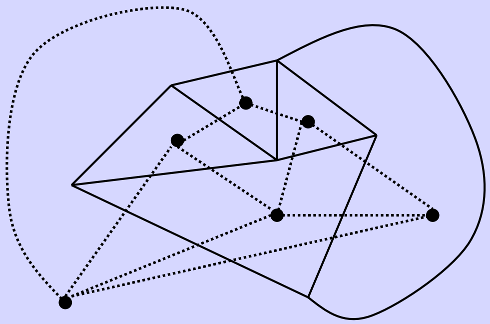
different frequencies for radio stations that interfere with each other?

different colors to color a map?

Map Coloring



Countries are the Vertices



Planar Four Coloring

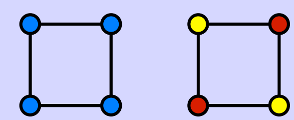
any planar map is 4-colorable.
1850's: false proof published
(was correct for 5 colors).
1970's: proof with computer
1990's: much improved

Chromatic Number

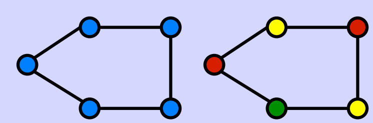
min #colors for G is chromatic number

$$\chi(G)$$

Simple Cycles

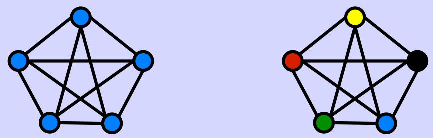


$\chi(C_{even}) = 2$



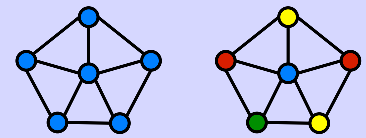
$\chi(C_{odd}) = 3$

Complete Graph K_5



$$\chi(K_n) = n$$

The Wheel W_n



W_5

$$\chi(W_{odd}) = 4$$
$$\chi(W_{even}) = 3$$

Bounded Degree

all degrees $\leq k$, implies

$$\chi(G) \leq k+1$$

very simple algorithm…

"Greedy" Coloring

…color vertices in any order.
next vertex gets a color different from its neighbors.

$\leq k$ neighbors, so
$k+1$ colors always work

coloring arbitrary graphs

2-colorable? --easy to check

3-colorable? --hard to check (even if planar)

find $\chi(G)$? --theoretically no harder than 3-color, but harder in practice