

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Simple Graphs: Coloring



Albert R Meyer, April 5, 2013

coloring.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Flight Gates



flights need gates, but  
times overlap.

how many gates needed?

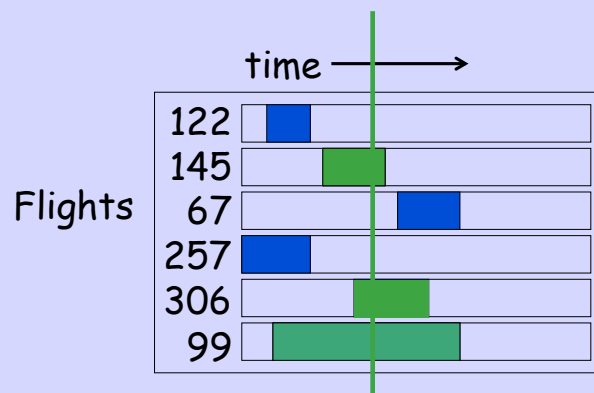


Albert R Meyer, April 5, 2013

coloring.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Airline Schedule

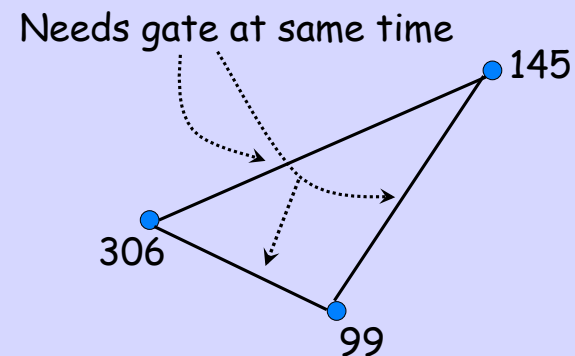


Albert R Meyer, April 5, 2013

coloring.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Conflicts Among 3 Flights

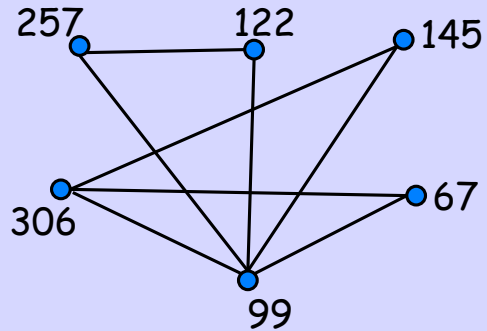


Albert R Meyer, April 5, 2013

coloring.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Model all Conflicts with a Graph



Albert R Meyer, April 5, 2013

coloring.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Color the vertices



Color vertices so that adjacent vertices have different colors.  
 min # distinct colors needed =  
 min # gates needed

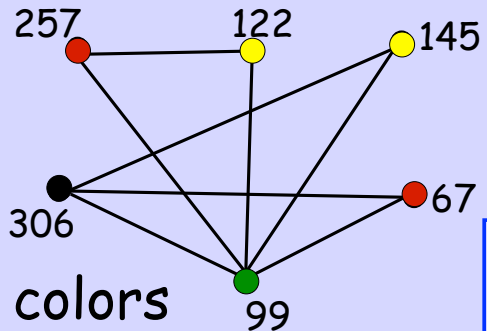


Albert R Meyer, April 5, 2013

coloring.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Coloring the Vertices



assign gates:

- 257, 67
- 122, 145
- 99
- 306

4 colors  
4 gates

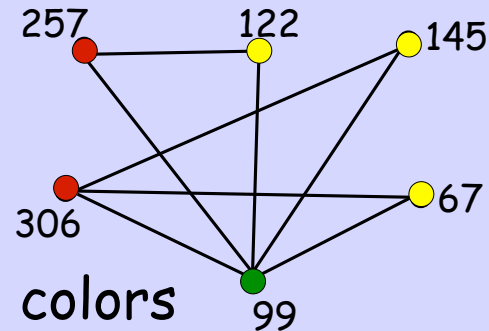


Albert R Meyer, April 5, 2013

coloring.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Better coloring



3 colors  
3 gates



Albert R Meyer, April 5, 2013

coloring.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Final Exams

subjects **conflict** if student takes both, so need different time slots.  
**how short** an exam period?

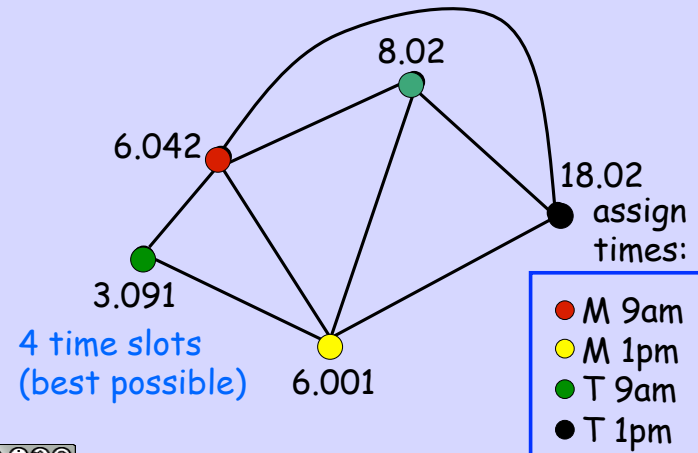


Albert R Meyer, April 5, 2013

coloring.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Model as a Graph



Albert R Meyer, April 5, 2013

coloring.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Conflicting Allocation Problems

- # separate **habitats** to house different species of animals, some **incompatible** with others?
- # different **frequencies** for radio stations that **interfere** with each other?
- # different colors to **color a map**?

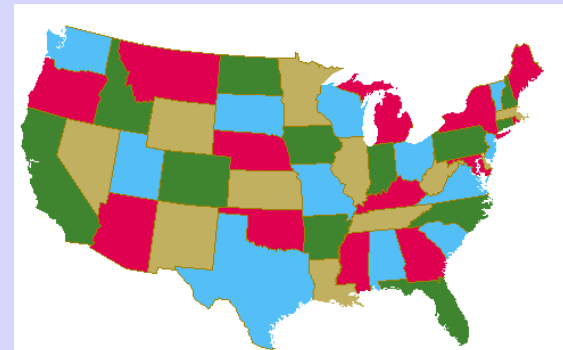


Albert R Meyer, April 5, 2013

coloring.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Map Coloring

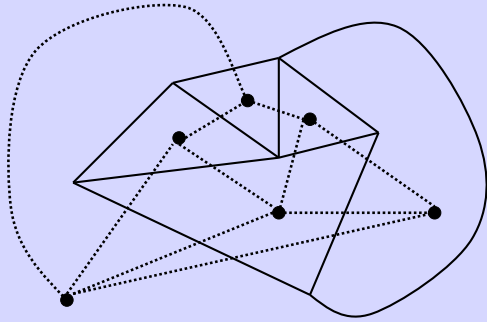


Albert R Meyer, April 5, 2013

coloring.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Countries are the Vertices



Albert R Meyer, April 5, 2013

coloring.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Planar Four Coloring

any planar map is 4-colorable.  
 1850's: false proof published  
 (was correct for 5 colors).  
 1970's: proof with computer  
 1990's: much improved



Albert R Meyer, April 5, 2013

coloring.14

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Chromatic Number

min #colors for  $G$  is  
 chromatic number

$$\chi(G)$$

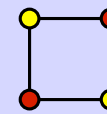
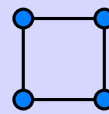


Albert R Meyer, April 5, 2013

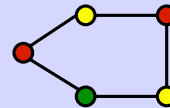
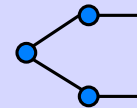
coloring.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Simple Cycles



$$\chi(C_{\text{even}}) = 2$$



$$\chi(C_{\text{odd}}) = 3$$

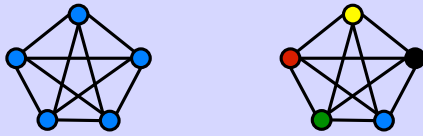


Albert R Meyer, April 5, 2013

coloring.18

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Complete Graph $K_5$



$$\chi(K_n) = n$$

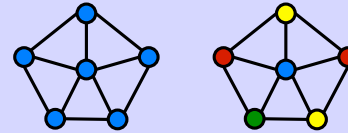


Albert R Meyer, April 5, 2013

coloring.19

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## The Wheel $W_n$



$W_5$

$$\chi(W_{\text{odd}}) = 4$$

$$\chi(W_{\text{even}}) = 3$$



Albert R Meyer, April 5, 2013

coloring.20

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Bounded Degree

all degrees  $\leq k$ , implies

$$\chi(G) \leq k+1$$

very simple algorithm...



Albert R Meyer, April 5, 2013

coloring.21

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## "Greedy" Coloring

...color vertices in any order.  
next vertex gets a color  
different from its neighbors.

$\leq k$  neighbors, so  
 $k+1$  colors always work



Albert R Meyer, April 5, 2013

coloring.22

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## coloring arbitrary graphs

2-colorable? --easy to check

3-colorable? --hard to check  
(even if planar)

find  $\chi(G)$ ? --theoretically  
no harder than 3-color, but  
harder in practice

