

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science

MIT 6.042J/18.062J

Sets: definitions



Albert R Meyer

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set-defs.1

6	9	13	7
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What is a Set?

Informally:

A **set** is a collection of mathematical objects, with the collection treated as a single mathematical object.

(This is **circular** of course:
what's a **collection**?)



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set-defs.2

6	9	13	7
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Familiar sets

real numbers \mathbb{R}

complex numbers \mathbb{C}

integers \mathbb{Z}

empty set \emptyset



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set-defs.3

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A set of 4 things

$\{7, \text{"Albert R."}, \pi/2, \mathbb{T}\}$

A set with 4 **elements**: two numbers, a string, and a Boolean.

Same as

$\{\mathbb{T}, \text{"Albert R."}, 7, \pi/2\}$

-- order doesn't matter



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set-defs.4

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In or Not In

An element is **in** or **not in** a set:
 $\{7, \pi/2, 7\}$ is same as $\{7, \pi/2\}$
No notion of being in the set
more than once.



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set-defs.5

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Membership

x is a **member** of A : $x \in A$

$\pi/2 \in \{7, \text{"Albert R."}, \pi/2, \top\}$

$14/2 \in$

$\pi/3 \notin$



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set-defs.6

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Synonyms for Membership

$x \in A$

x is an **element** of A

x is **in** A

examples:

$7 \in \mathbb{Z}$, $2/3 \notin \mathbb{Z}$, $\mathbb{Z} \in \{\top, \mathbb{Z}, 7\}$



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set-defs.7

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Subset (\subseteq)

$A \subseteq B$ A is a **subset** of B
 A is **contained in** B

Every element of A is also
 an element of B :

$\forall x [x \in A \text{ IMPLIES } x \in B]$



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set-defs.8

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Subset

examples:

$$\mathbb{Z} \subseteq \mathbb{R}, \quad \mathbb{R} \subseteq \mathbb{C}, \quad \{3\} \subseteq \{5, 7, 3\}$$

$$A \subseteq A, \quad \emptyset \subseteq \text{every set}$$



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set-defs.9

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$\emptyset \subseteq$ everything

$\emptyset \subseteq B$ is defined to mean

$$\forall x [x \in \emptyset \text{ IMPLIES } x \in B]$$

false

true



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set-defs.10

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Defining Sets

The set of elements x in A
such that $P(x)$ is true.

$$\{x \in A \mid P(x)\}$$



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set-defs.11

6	9	13	7
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Defining Sets

The set of elements x in A
such that $P(x)$ is true.

$$\{x \in A \mid P(x)\}$$



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set-defs.12

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Defining Sets

The set E of even integers:

$$\{n \in \mathbb{Z} \mid n \text{ is even}\}$$



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set-defs.13

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Power Set

$\text{pow}(A) ::=$ all the subsets of A
 $= \{B \mid B \subseteq A\}$

example:

$$\text{pow}(\{T, F\}) = \{\{T\}, \{F\}, \{T, F\}, \emptyset\}$$

$$E \in \text{pow}(\mathbb{Z}), \quad \mathbb{Z} \in \text{pow}(\mathbb{R})$$

$$B \in \text{pow}(A) \text{ IFF } B \subseteq A$$



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set-defs.14