

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Relations & Functions



Albert R. Meyer

February 21, 2011

lec 3T.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary relations

A **binary relation** associates elements of one set called the **domain**, with elements of another set called the **codomain**



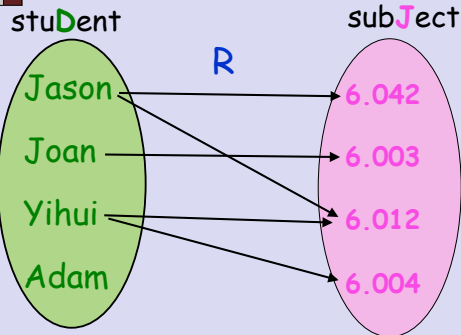
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"Registered for" relation R



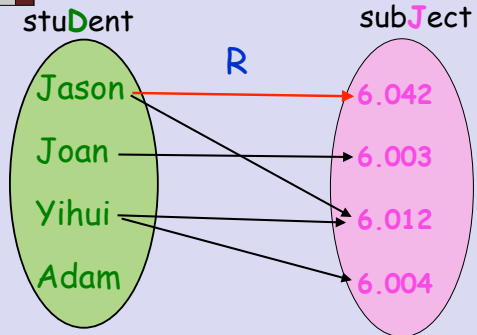
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"Registered for" relation R



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"Registered for" relation R

Jason is registered for 6.042

notation:

Jason R 6.042 infix

$R(\text{Jason}, 6.042)$ prefix

$(\text{Jason}, 6.042) \in R$

$(\text{Jason}, 6.042) \in \text{graph}(R)$



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

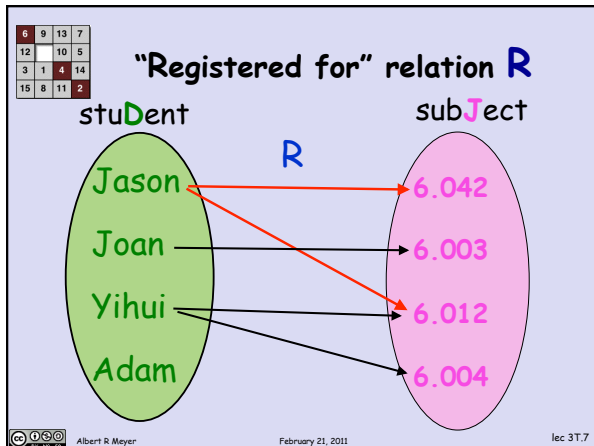
$R(\text{Jason}) =$ subjects Jason is registered for



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lec 3T.6



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\text{Jason}) = \text{subjects Jason is registered for}$
 $= \{6.042, 6.012\}$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::= \text{all the subjects being taken by students in the set } X$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::= \text{everything } R \text{ relates to things in } X$

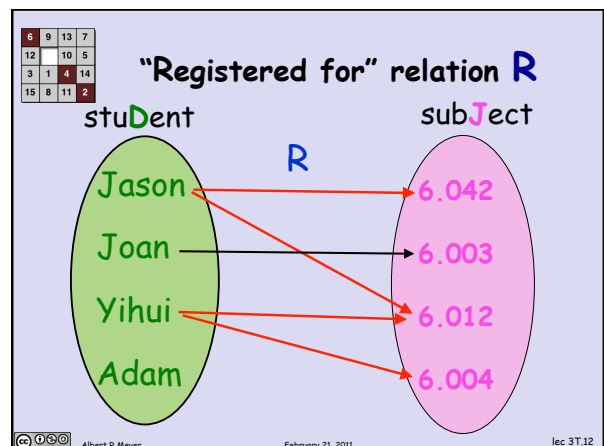
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lec 3T.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\{\text{Jason, Yihui}\}) = \text{subjects with Jason or Yihui registered}$

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lec 3T.11



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(\{\text{Jason}, \text{Yihui}\}) =$
 subjects with Jason
 or Yihui registered
 $= \{6.042, 6.012, 6.004\}$



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R

$R(X) ::=$ endpoints of
 arrows from points in X

$$\{j \in J \mid \exists d \in X. d R j\}$$

an arrow from X goes to j



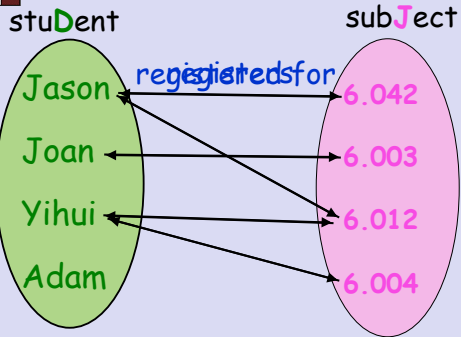
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"registers" relation R^{-1}



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"registers" relation R^{-1}

$$d R j \quad \text{IFF} \quad j R^{-1} d$$



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$$R^{-1}(6.012) =$$



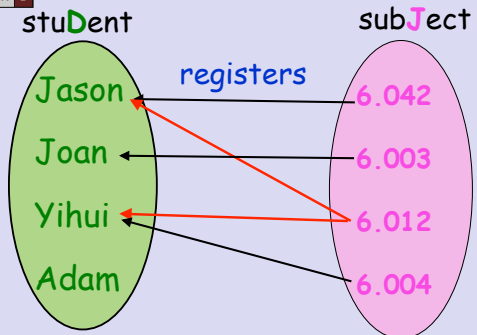
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lec 3T.17

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"registers" relation R^{-1}



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$$R^{-1}(6.012) = \{\text{Jason}, \text{Yihui}\}$$



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$$R^{-1}(6.012) = \{\text{Jason}, \text{Yihui}\}$$

$$R^{-1}(\{6.012, 6.003\}) =$$



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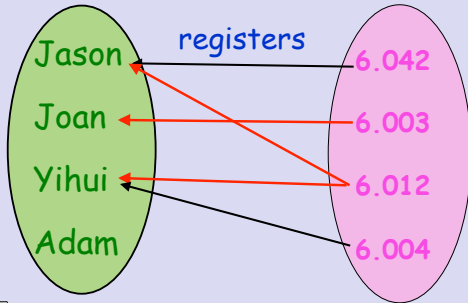
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"registers" relation R^{-1}

stuDent subJect



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Images under R^{-1}

$$R^{-1}(6.012) = \{\text{Jason}, \text{Yihui}\}$$

$$R^{-1}(\{6.012, 6.003\}) =$$

$$\{\text{Jason}, \text{Joan}, \text{Yihui}\}$$

$R^{-1}(Y)$ aka the **inverse image**
of Y under R



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Inverse image under R

$R^{-1}(J) =$ all the stuDents registered

for some subJect

Every student is registered

for some subject:

$$D \subseteq R^{-1}(J)$$

(not true: Adam wasn't registered)



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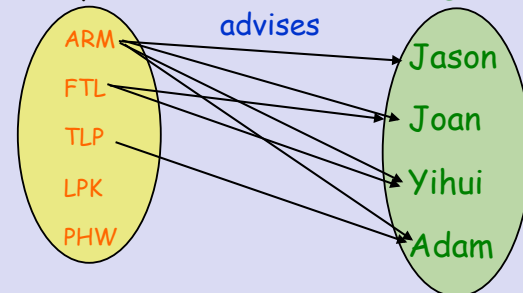
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

"advises" relation V

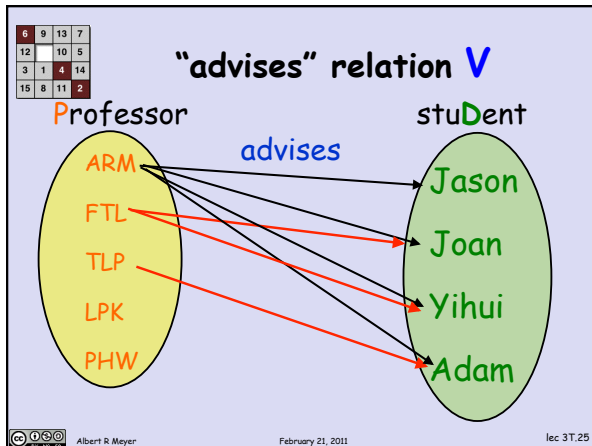
Professor stuDent



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Composing R and V

$$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\})$$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Composing R and V

$$R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\})$$

$$= \{6.003, 6.012, 6.004\}$$

$R(V(X))$ = subjects that advisees of
profs in X are registered for

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Composing R and V

$$(R \circ V)(X) ::= R(V(X))$$

R \circ V

is the composition
of R and V

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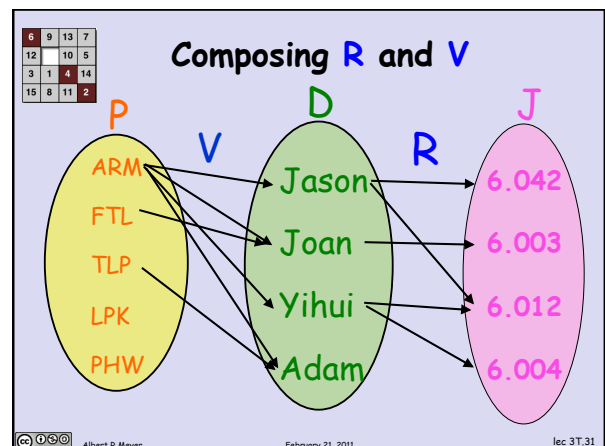
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

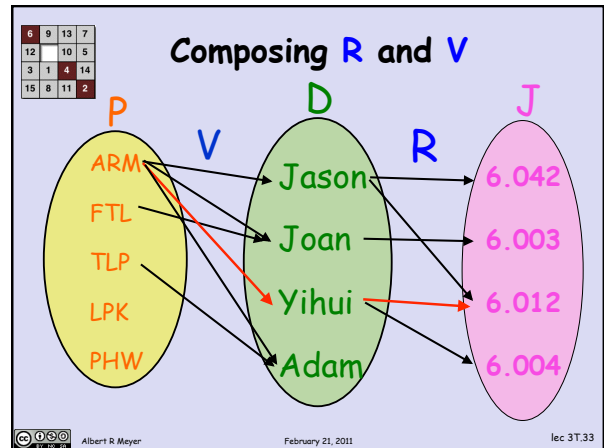
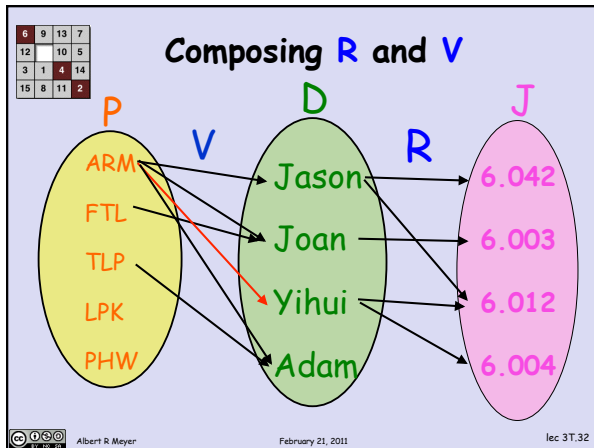
Composing R and V

$R \circ V$::= "prof has advisee registered for"

$p(R \circ V)j$::= prof p has an advisee registered in subject j

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6 9 13 7
12 10 5
3 1 4 14
15 8 11 2

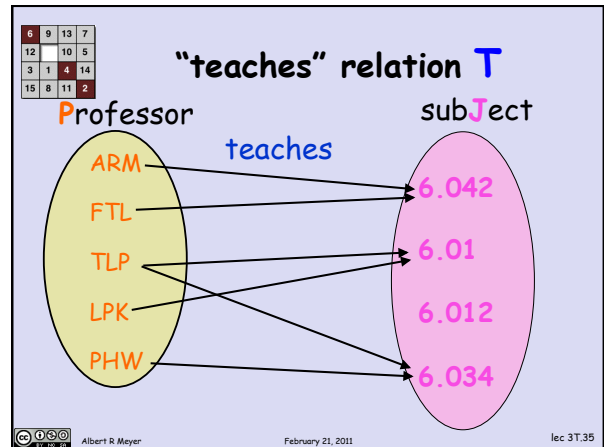
Composing R and V

ARM (R ∘ V) 6.012 because
 ARM V Yihui AND Yihui R 6.012

$$p(R \circ V)j \text{ IFF } \exists d \in D. [pVd \text{ AND } dRj]$$

note: V, R in reverse order

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6 9 13 7
12 10 5
3 1 4 14
15 8 11 2

set operations on relations

Profs should not teach their advisees:

$$\forall p \forall j. \text{NOT}(p(R \circ V)j \text{ AND } pTj)$$

$$T \cap (R \circ V) = \emptyset$$

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6 9 13 7
12 10 5
3 1 4 14
15 8 11 2

set operations on relations

Profs should not teach their advisees:

$$\forall p \forall j. \text{NOT}(p(R \circ V)j \text{ AND } pTj)$$

$$R \circ V \subseteq \overline{T}$$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary relations

A binary relation, R , from a set A to a set B associates elements of A with elements of B .



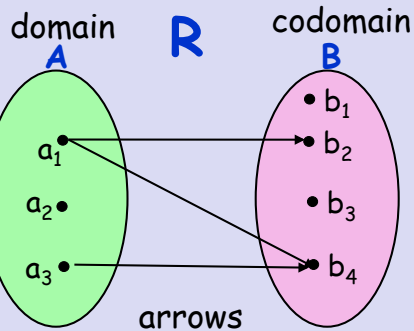
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary relation R from A to B



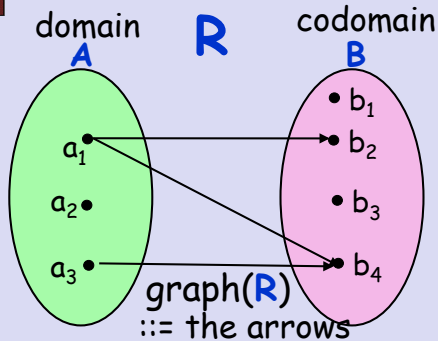
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary relation R from A to B



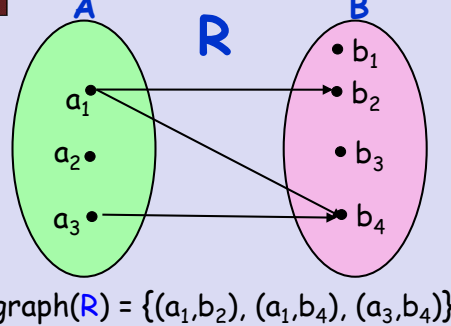
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary relation R from A to B



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

range(R)

range(R) ::= elements with arrows coming in
= $R(A)$



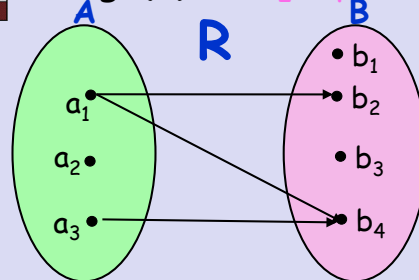
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

range(R) = $\{b_2, b_4\}$



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Functions are relations

A function, F , from A to B is a relation which associates each element, a , of A with at most one element of B .
 called $F(a)$



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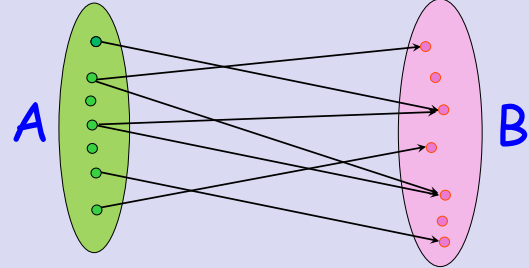
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

function archery

≤ 1 arrow out



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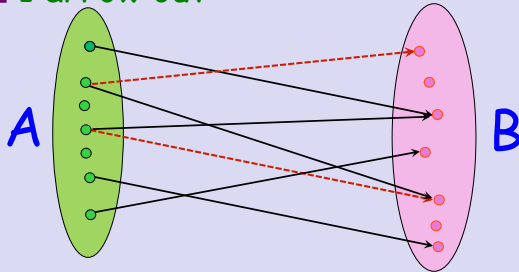
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

function archery

≤ 1 arrow out



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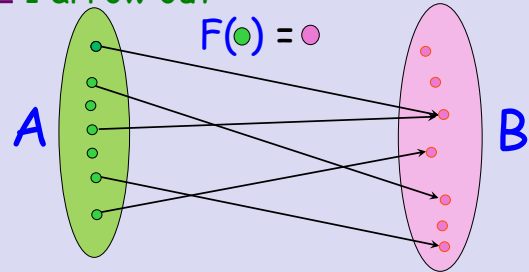
lec 3T.48

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

function archery

≤ 1 arrow out

$$F(\bullet) = \bullet$$



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lec 3T.49

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Functions are relations

relation $F:A \rightarrow B$ is a function

IFF $|F(a)| \leq 1$

IFF

$[aFb \text{ AND } aFb'] \text{ IMPLIES } b = b'$



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