Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Number of Heads

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n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[\#\ \text{Heads}] = E[B_{n,p}]$$

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Expected #Heads

Binomial theorem and differentiating gives a closed formula

Binomial Expectation

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

take $\partial / \partial x$:

$$n(x+y)^{n-1} = \frac{1}{x} \sum_{k=0}^{n} k \binom{n}{k} x^k y^{n-k}$$

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$$n \qquad = \frac{1}{p} E\left[B_{n,p}\right]$$

$$\boxed{np = E\left[B_{n,p}\right]}$$