

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Number of Heads



Albert R Meyer,

May 8, 2013

lec 12F.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[\# \text{ Heads}] = E[B_{n,p}]$$



Albert R Meyer,

May 8, 2013

lec 12F.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Expected #Heads

Binomial theorem and differentiating gives a closed formula



Albert R Meyer,

May 8, 2013

lec 12F.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

take $\partial / \partial x$:

$$n(x + y)^{n-1} = \frac{1}{x} \sum_{k=0}^n k \binom{n}{k} x^k y^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n(x + y)^{n-1} = \frac{1}{x} \sum_{k=0}^n k \binom{n}{k} x^k y^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n(p + q)^{n-1} = \frac{1}{p} \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n = \frac{1}{p} \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$



Albert R Meyer,

May 8, 2013

lec 12F.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n = \frac{1}{p} E[B_{n,p}]$$

$$np = E[B_{n,p}]$$



Albert R Meyer,

May 8, 2013

lec 12F.11