

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Time to Failure



Albert R Meyer,

May 8, 2013

ranvarfail.1

6	9	13	7
12		10	5
3	1	4	14
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Mean Time to "Failure"

Flip a coin until a **Head** comes up

$$\Pr[\text{Head}] = p$$

$F ::= \# \text{flips to 1st Head}$

$$E[F] ?$$



Albert R Meyer,

May 8, 2013

ranvarfail.3

6	9	13	7
12		10	5
3	1	4	14
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Mean Time to "Failure"

$$\Pr[F=1] = \Pr[H] = p$$



Albert R Meyer,

May 8, 2013

ranvarfail.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$\Pr[F=1] = \Pr[H] = p$$

$$\Pr[F=2] = \Pr[TH] = q \cdot p$$

$$\Pr[F=3] = \Pr[TTH] = q^2 \cdot p$$

$$\text{PDF}_F(n) = q^{n-1} p$$

Geometric Distribution



Albert R Meyer,

May 8, 2013

ranvarfail.5

6	9	13	7
12	10	5	
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Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \underbrace{\sum_{n \geq 0} (n+1) q^n}_{\frac{1}{(1-q)^2}}
 \end{aligned}$$



Albert R Meyer,

May 8, 2013

ranvarfail.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \frac{1}{(1-q)^2}
 \end{aligned}$$



Albert R Meyer,

May 8, 2013

ranvarfail.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$\begin{aligned}
 E[F] &= \sum_{n>0} n \cdot \Pr[F=n] \\
 &= \sum_{n>0} n \cdot q^{n-1} p \\
 &= p \frac{1}{p^2} = \frac{1}{p}
 \end{aligned}$$



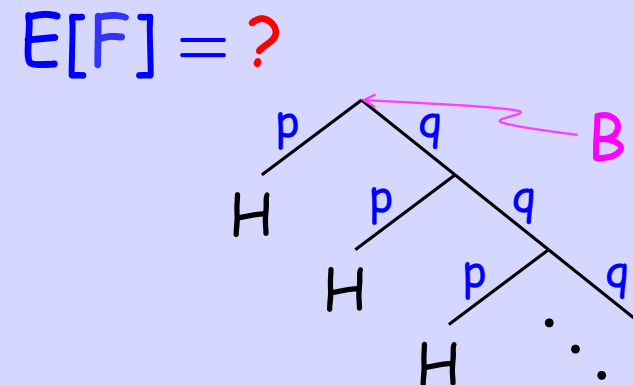
Albert R Meyer,

May 8, 2013

ranvarfail.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"



Albert R Meyer,

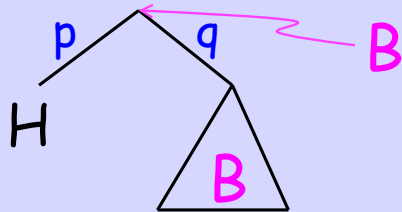
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ranvarfail.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$E[F] = ?$$



now use Total Expectation



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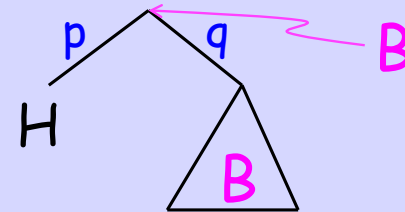
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ranvarfail.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$E[F] =$$



$$E[F | 1^{\text{st}} \text{ is } H] \cdot p$$



Albert R Meyer,

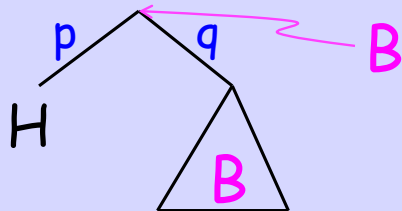
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ranvarfail.13

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$E[F] =$$



$$\underbrace{E[F | 1^{\text{st}} \text{ is } H] \cdot p}_1 + \underbrace{E[F | 1^{\text{st}} \text{ is } T] \cdot q}_{E[F+1]}$$



Albert R Meyer,

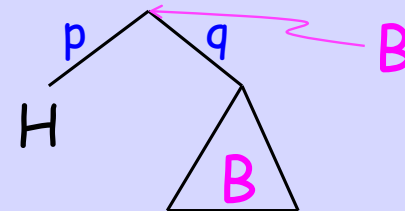
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ranvarfail.14

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$E[F] =$$



$$p + (E[F] + 1) \cdot q$$

now solve for $E[F]$



Albert R Meyer,

May 8, 2013

ranvarfail.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mean Time to "Failure"

$$E[F] =$$

$$\frac{1}{p}$$



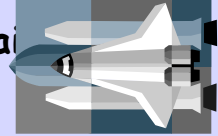
Albert R Meyer,

May 8, 2013

ranvarfail.16

6	9	13	7
12	10	5	
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Mean Time to Failure



application: if space station Mir has $1/150,000$ chance of destruction in any given hour, how many hours expected until destruction?

$150,000$ hours \approx 17 years



Albert R Meyer,

May 8, 2013

ranvarfail.17

6	9	13	7
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Intuitive argument

$$E[\text{\#fails in 1 try}] = p$$

$$E[\text{\#fails in } n \text{ tries}] = np$$

$$E[\text{\#tries between fails}]$$

$$= \frac{\text{\# tries}}{\text{\# fails}} = \frac{n}{np} = \frac{1}{p}$$



Albert R Meyer,

May 8, 2013

ranvarfail.18