Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Time to Failure

Mean Time to "Failure"

Flip a coin until a Head comes up

Pr[Head] = p

F ::= #flips to 1st Head

E[F] ?

Mean Time to "Failure"

Pr[F= 1] = Pr[H] = p

Mean Time to "Failure"

Pr[F= 1] = Pr[H] = p

Pr[F= 2] = Pr[TH] = $q \cdot p$

Pr[F= 3] = Pr[TTH] = $q^2 \cdot p$

$$PDF_F(n) = q^{n-1}p$$

Geometric Distribution

Mean Time to "Failure"

$$E[F] = \sum_{n>0} n \cdot \Pr[F=n] = \sum_{n>0} n \cdot q^{n-1} p = p \underbrace{\sum_{n \geq 0} (n+1) q^n}_{\frac{1}{(1-q)^2}}$$

Mean Time to "Failure"

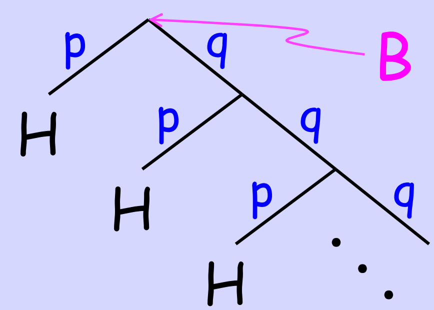
$$E[F] = \sum_{n>0} n \cdot \Pr[F=n] = \sum_{n>0} n \cdot q^{n-1} p = p \frac{1}{(1-q)^2}$$

Mean Time to "Failure"

$$\boxed{E[F]} = \sum_{n>0} n \cdot \Pr[F=n] = \sum_{n>0} n \cdot q^{n-1} p = p \frac{1}{p^2} \boxed{= \frac{1}{p}}$$

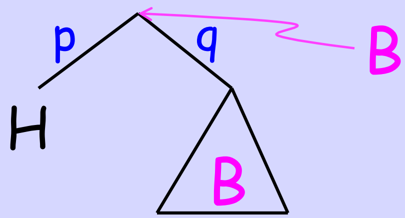
Mean Time to "Failure"

$$E[F] = ?$$



Mean Time to "Failure"

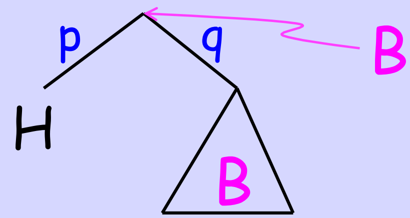
$E[F] = ?$



now use Total Expectation

Mean Time to "Failure"

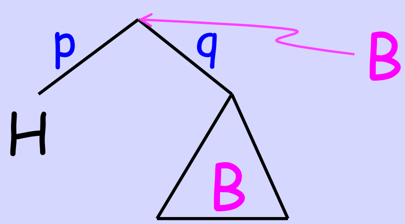
$E[F] =$



$E[F \mid 1^{st} \text{ is } H] \cdot p$

Mean Time to "Failure"

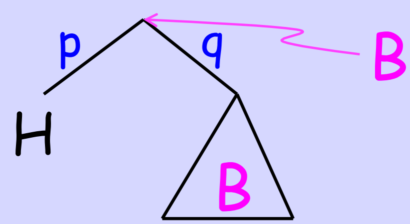
$E[F] =$



$$\underbrace{E[F \mid 1^{st} \text{ is } H]}_{1} \cdot p + \underbrace{E[F \mid 1^{st} \text{ is } T]}_{E[F+1]} \cdot q$$

Mean Time to "Failure"

$E[F] =$



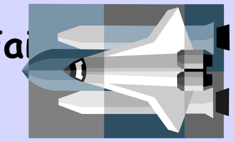
$p + (E[F] + 1) \cdot q$

now solve for $E[F]$

Mean Time to "Failure"

$$E[F] = \frac{1}{p}$$

Mean Time to Fai



application: if space station Mir has 1/150,000 chance of destruction in any given hour, how may hours expected until destruction?

150,000 hours $\approx$ 17 years

Intuitive argument

$E[\#\text{fails in 1 try}] = p$

$E[\#\text{fails in n tries}] = np$

$E[\#\text{tries between fails}]$

$$= \frac{\#\text{tries}}{\#\text{fails}} = \frac{n}{np} = \frac{1}{p}$$