Problem Set 7

Due: April 27

Reading: Chapter 14. *Sums and Asymptotics* through 14.4. *Book Stacking*, and 14.7. *Asymptotics*

Problem 1. (a) In the proof of Lemma 14.7.3 that $\log_2 x = o(x^\epsilon)$ for any $\epsilon > 0$, we relied on the "familiar fact" that $\log_2 x < x$ for all $x > 1$. Prove this familiar fact (may require elementary calculus).

(b) Prove that the relation R on positive functions such that $f \; R \; g$ iff $g = o(f)$ is a strict partial order.

(c) If g is a positive function, prove than $f \sim g$ iff $f = g + h$ for some function $h = o(g)$.

Problem 2.
Use integration to find upper and lower bounds that differ by at most 0.1 for the following sum. (You may need to add the first few terms explicitly and then use integrals to bound the sum of the remaining terms.)

$$\sum_{i=1}^{\infty} \frac{1}{(2i+1)^2}$$

Problem 3.
Is a Harvard degree really worth more than an MIT degree? Let us say that a person with a Harvard degree starts with \$40,000 and gets a \$20,000 raise every year after graduation, whereas a person with an MIT degree starts with \$30,000, but gets a 20% raise every year. Assume inflation is a fixed 8% every year. That is, \$1.08 a year from now is worth \$1.00 today.

(a) How much is a Harvard degree worth today if the holder will work for n years following graduation?

(b) How much is an MIT degree worth in this case?

(c) If you plan to retire after twenty years, which degree would be worth more?

Hint: To check your formulas, your numbers in part (c) for $n = 20$ years should both be in the vicinity of \$2 million.