Problem Set 2

Due: March 2

Reading:

- Chapter 3.6. *Predicate Formulas*,
- Chapter 2. *The Well Ordering Principle* through 2.3 (omit 2.4. *Well Ordered Sets*),
- Chapter 4. *Mathematical Data Types*,
- Chapter 5. *Induction*.

Problem 1.
Use the Well Ordering Principle to prove that any integer greater than or equal to 50 can be represented as the sum of nonnegative integer multiples of 7, 11, and 13.

Hint: Use the template for WOP proofs to ensure partial credit. Verify that integers in the interval [50..56] are sums of nonnegative integer multiples of 7, 11, and 13.

Problem 2.
Let $R : A \to B$ and $S : B \to C$ be binary relations such that $S \circ R$ is a bijection and $|A| = 2$.

Give an example of such R, S where neither R nor S is a function. Indicate exactly which properties—total, surjection, function, and injection—your examples of R and S have.

Hint: Let $|B| = 4$.

Problem 3.
We examine a series of propositional formulas $F_1, F_2, \ldots, F_n, \ldots$ containing propositional variables $P_1, P_2, \ldots, P_n, \ldots$ constructed as follows

$$
\begin{array}{lcl}
F_1(P_1) & ::= & P_1 \\
F_2(P_1, P_2) & ::= & P_1 \text{ IMPLIES } P_2 \\
F_3(P_1, P_2, P_3) & ::= & (P_1 \text{ IMPLIES } P_2) \text{ IMPLIES } P_3 \\
F_4(P_1, P_2, P_3, P_4) & ::= & ((P_1 \text{ IMPLIES } P_2) \text{ IMPLIES } P_3) \text{ IMPLIES } P_4 \\
F_5(P_1, P_2, P_3, P_4, P_5) & ::= & (((P_1 \text{ IMPLIES } P_2) \text{ IMPLIES } P_3) \text{ IMPLIES } P_4) \text{ IMPLIES } P_5 \\
 & \vdots &
\end{array}
$$

Let T_n be the number of different true/false settings of the variables $P_1, P_2, \ldots, P_n$ for which the statement $F_n(P_1, P_2, \ldots, P_n)$ is true. For example, $T_2 = 3$ since $F_2(P_1, P_2)$ is true for 3 different settings of the variables P_1 and P_2:

P_1	P_2	$F_2(P_1, P_2)$
T	T	T
T	F	F
F	T	T
F	F	T

(a) Explain why

$$T_{n+1} = 2^{n+1} - T_n. \tag{1}$$

(b) Use induction to prove that

$$T_n = \frac{2^{n+1} + (-1)^n}{3} \tag{*}$$

for $n \geq 1$.