

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

Propositions & Logical Operations



Albert R Meyer

February 11, 2015

propositional ops.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Propositional (Boolean) Logic

A **proposition** is either **True** or **False**
Example:

There are 6 regular solids.

False

Non-examples:

Wake up!

Where am I?

It's 3PM.



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propositional ops.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

English to Math

Greeks carry Swords or Javelins

$$G \rightarrow (S \vee J)$$

True even if a Greek carries *both* a Sword and a Javelin



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propositional ops.3

6	9	13	7
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English to Math

Greeks carry Bronze or Copper swords

$$G \rightarrow (B \oplus C)$$

Bronze or Copper but **not both**



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propositional ops.4

6	9	13	7
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Definition of OR

The value of (P OR Q) is T iff P is T, or Q is T, or both are T.

Truth Table for OR

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

F iff both P, Q are F



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propositional ops.5

6	9	13	7
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Definition of XOR

The value of (P XOR Q) is T iff exactly one of P and Q is T.

Truth Table for XOR

P	Q	P XOR Q
T	T	F
T	F	T
F	T	T
F	F	F



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propositional ops.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Definition of AND

The value of (P AND Q) is T iff both P and Q are T.

Truth Table for AND

P	Q	P AND Q
T	T	T
T	F	F
F	T	F
F	F	F

T iff both P, Q are T



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propositional ops.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Definition of NOT

The value of NOT(P) is T iff the value of P is F.

Truth Table for NOT(P)

P	NOT(P)
T	F
F	T



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propositional ops.8

6	9	13	7
12		10	5
3	1	4	14
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Other Applications

Java Logical Expressions:

```
                OR      AND
if ((x>0) || (x <= 0 && y>100))
    :
    (more code)
```



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propositional ops.9

6	9	13	7
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Digital Logic

$\bar{x} ::= \text{NOT}(x)$

$1 ::= \text{T}$

$0 ::= \text{F}$

$\cdot ::= \text{AND}$

$+ ::= \text{OR}$



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propositional ops.10