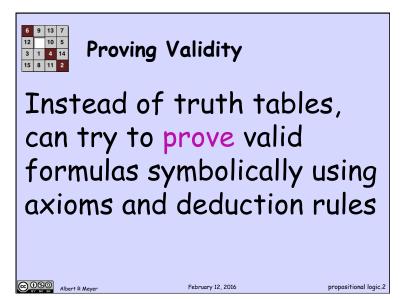
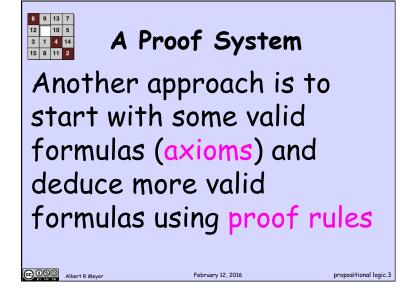
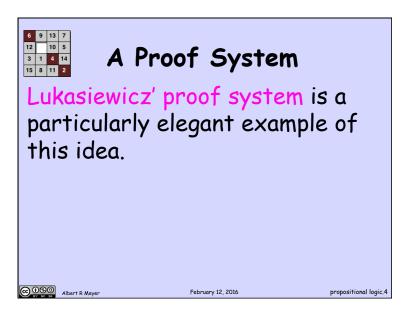


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propositional logic,1









### A Proof System

Lukasiewicz' proof system is a particularly elegant example of this idea. It covers formulas whose only logical operators are IMPLIES  $(\rightarrow)$  and NOT.

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## 6 9 13 7 12 10 5 3 1 4 14 15 8 11 2 Lukasiewicz' Proof System

#### Axioms:

- 1)  $(\neg P \rightarrow P) \rightarrow P$
- 2)  $P \rightarrow (\neg P \rightarrow Q)$
- 3)  $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

The only rule: modus ponens



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# Lukasiewicz' Proof System

Three Axioms:

$$(\overline{P} \to P) \to P$$
 $P \to (\overline{P} \to Q)$ 
 $(P \to Q) \to ((Q \to R) \to (P \to R))$ 

One Rule: modus ponens



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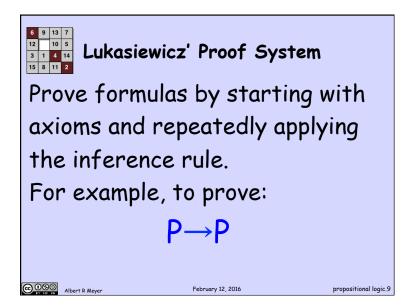
### 12 10 5 3 1 4 14 15 8 11 2 Lukasiewicz' Proof System

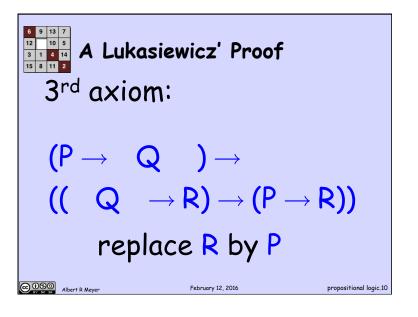
Prove formulas by starting with axioms and repeatedly applying the inference rule.

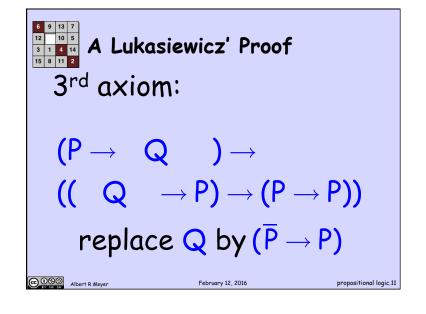
To illustrate the proof system we'll do an example, which you may safely skip.

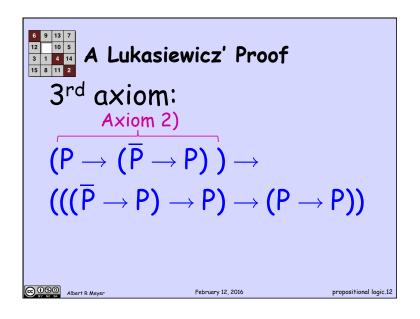


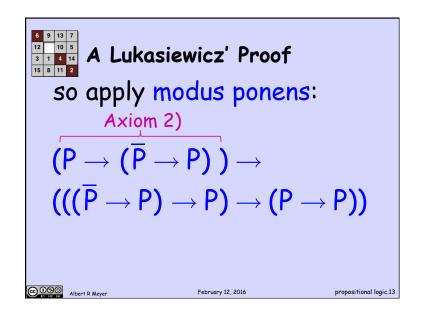
propositional logic.8

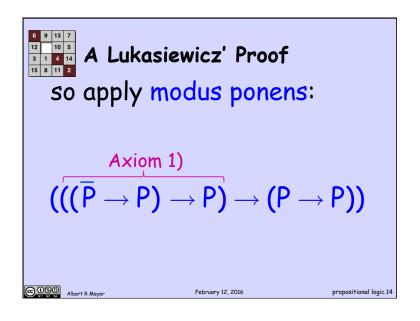


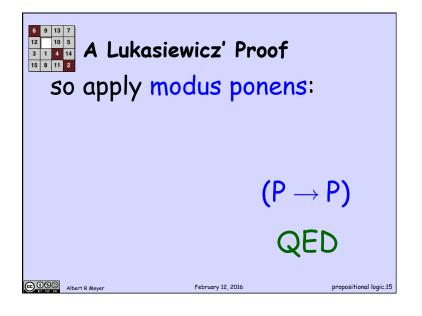


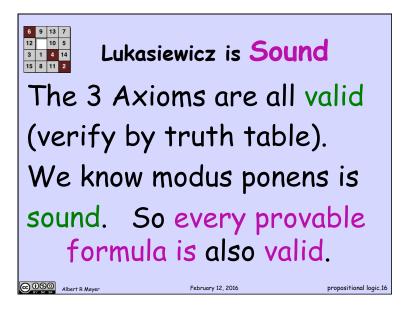


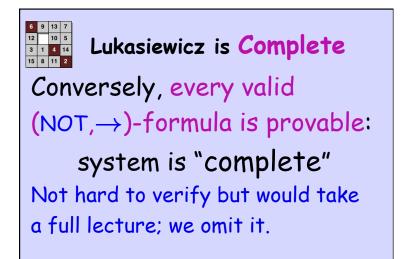












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