

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mathematics for Computer Science
6.042J/18.062J

PROOFS, II



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proof-intro.II.1

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Another **Bogus** Proof

Theorem: Every polynomial,
 $ax^2 + bx + c$
has two roots over \mathbb{C} .

Proof (by calculation). The roots are:

$$r_1 ::= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 ::= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$



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proof-intro.II.2

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Another **Bogus** Proof

Counter-examples:

$0x^2 + 0x + 1$ has 0 roots

$0x^2 + 1x + 1$ has 1 root

The **bug**: divide by zero error

The **fix**: require $a \neq 0$



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proof-intro.II.3

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Another **Bogus** Proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The **bug**: $r_1 = r_2$

The **fix**: require $D \neq 0$ where

$$D ::= b^2 - 4ac$$



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proof-intro.II.4

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Another **Bogus** Proof

What if $D < 0$?

$x^2 + 1$ has roots $i, -i$

--**ambiguous** which is r_1
and which is r_2 ?



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proof-intro.II.5

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

$$1 = -1 ?$$

ambiguity can cause problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$$

Moral:

1. Be sure rules are properly applied.
2. Thoughtless calculation no substitute for understanding.



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proof-intro.II.6

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Consequences of $1 = -1$

$$\frac{1}{2} = -\frac{1}{2} \quad (\text{multiply by } \frac{1}{2})$$

$$2 = 1 \quad (\text{add } \frac{3}{2})$$

*“Since I and the Pope are clearly 2,
we conclude that I and the Pope are 1.
That is, I am the Pope.”*

-- Bertrand Russell



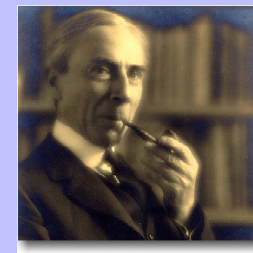
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proof-intro.II.8

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Consequences of $1 = -1$



Bertrand Russell (1872 - 1970)

(Picture source: <http://www.users.drew.edu/~jlanz/brs.html>)



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proof-intro.II.9