Mathematics for Computer Science
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Independent Events

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Definition 1:

Events A and B are independent iff

$$\Pr[A] = \Pr[A \mid B]$$

Definition 2:

Events A and B are independent iff

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$

Definitions of Independence

proof of equivalence:

$$\Pr[A] = \Pr[A \mid B] \quad \text{iff}$$

$$\Pr[A] = \frac{\Pr[A \cap B]}{\Pr[B]} \quad \text{iff}$$

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$

Definitions of Independence

need $\Pr[B] \neq 0$ for Def. 1.

Def. 2 always works:

$$\Pr[A\}\cdot \Pr[B] = \Pr[A \cap B]$$

Independence

$Pr[A] \cdot Pr[B] = Pr[A \cap B]$

symmetric in A and B so,

A independent of B iff

B independent of A

Independence

Corollary: If $Pr[B]= 0$, then B is independent of every event

Independence

Corollary: If $Pr[B]= 0$, then B is independent of every event – even itself.

Independence

A independent of B means

Independence

A independent of B means A is independent of whether or not B occurs:

Independence

Lemma:

A independent of B iff A independent of $\overline{B}$.

Independence

Lemma:

A independent of B iff A independent of $\overline{B}$

Simple proof using:

$\Pr[A-B] = \Pr[A]-\Pr[A\cap B]$