

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

The Pigeonhole Principle



Albert R Meyer, April 24, 2013

pigeonhole.1

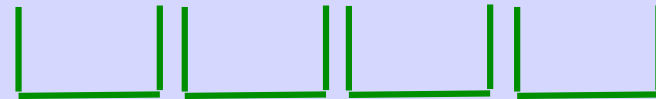
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Pigeonhole Principle

If **more** pigeons



than pigeonholes,



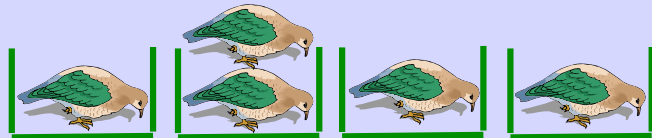
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pigeonhole.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Pigeonhole Principle

then **some hole** must
have \geq **two** pigeons!



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pigeonhole.3

6	9	13	7
12		10	5
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Pigeonhole Principle

Mapping Rule: **total injection** from
 A to B implies $|A| \leq |B|$.

If $|A| > |B|$, then
no total injection from A to B .



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pigeonhole.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

example: 5 Card Draw

set of 5 cards:
must have ≥ 2
with the same suit.



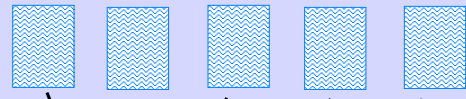
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pigeonhole.6

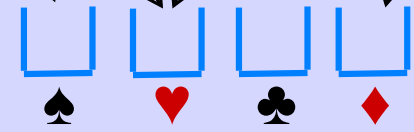
6	9	13	7
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5 Card Draw

5 cards
(pigeons)



4 suits
(holes)



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pigeonhole.7

6	9	13	7
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10 Card Draw

10 cards: how many have
the same suit?

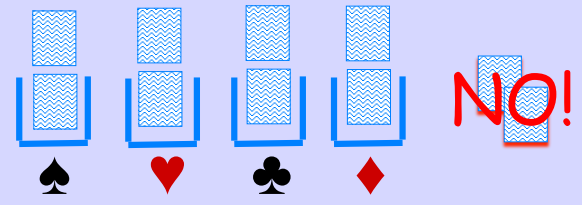


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pigeonhole.8

6	9	13	7
12		10	5
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15	8	11	2

10 Card Draw



< 3 cards in every hole?



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pigeonhole.9

6	9	13	7
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15	8	11	2

10 Card Draw

cards with same suit

$$\geq \left\lceil \frac{10}{4} \right\rceil = 3$$

"ceiling," means round up



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pigeonhole.10

6	9	13	7
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Generalized Pigeonhole Principle

If n pigeons and h holes,
then some hole has \geq

$$\left\lceil \frac{n}{h} \right\rceil \text{ pigeons.}$$



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pigeonhole.11