


6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
 MIT 6.042J/18.062J

Partial Orders



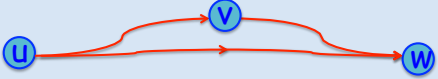
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po's.1


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Walks in digraph G

walk from u to v and
 from v to w



implies walk from u to w



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po's.2

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Walks in digraph G

walk from u to v and
 from v to w , implies
 walk from u to w :

uG^+v AND vG^+w
 IMPLIES uG^+w



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po's.3


6	9	13	7
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Walks in digraph G

transitive relation R :

uRv AND vRw
 IMPLIES uRw

G^+ is transitive



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po's.4


6	9	13	7
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transitivity

Theorem:

R is a transitive iff

$R = G^+$ for some digraph G


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Paths in DAG D

pos length path from u to v implies
no path from v to u

$u D^+ v$ IMPLIES NOT($v D^+ u$)

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Paths in DAG D

asymmetric relation R :

$u R v$ IMPLIES NOT($v R u$)


D^+ is asymmetric

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strict partial orders

transitive &
asymmetric

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strict partial orders

examples:

- \subset on sets
- "indirect prerequisite" on MIT subjects
- less than, $<$, on real numbers



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po's.9

6	9	13	7
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strict partial orders

Theorem:

R is a SPO iff

$R = D^+$ for some
DAG D



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po's.10

6	9	13	7
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linear orders

Given any two elements,
one will be "bigger than"
the other one.



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po's.11

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linear orders

basic example:

$<$ or \leq on the Reals:
if $x \neq y$, then either

$x < y$ OR $y < x$



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po's.12

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linear orders

\mathbb{R} is linear:
no incomparable elements


if $x \neq y$, then either
 $x R y$ OR $y R x$


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6	9	13	7
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linear orders

The whole partial order is a chain




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linear orders

A topological sort turns a partial order into a linear order ...in a way that is consistent with the partial order

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weak partial orders

same as a strict partial order R , except that $a R a$ always holds

examples:


- \leq is weak p.o. on \mathbb{R}
- \subseteq is weak p.o. on sets

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reflexivity


relation R on set A
is **reflexive** iff
 aRa for all $a \in A$
 G^* is reflexive

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antisymmetry

binary relation R is
antisymmetric iff
it is asymmetric
except for aRa case.


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A/Antisymmetry

minor difference:
whether aRa is allowed

never **sometimes**

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antisymmetry

antisymmetric relation R :

uRv IMPLIES **NOT**(vRu)
for $u \neq v$

D^* is antisymmetric for
DAG D

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weak partial orders

transitive,
antisymmetric &
reflexive



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po's.21

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weak partial orders

Theorem:

R is a **WPO** iff

$R = D^*$ for some
DAG D



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po's.22