Midterm Exam February 22

Your name: ______________________

Indicate your Identify your Team:	**1PM**	**32-082 Table (1–13):**	_______
	1PM	**32-044 Table (A–K):**	_______
	2:30PM	**32-044 Table (A–K):**	_______

- This exam is **closed book** except for a 2-sided cribsheet. Total time is 90 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem.
- In answering the following questions, you may use without proof any of the results from class or text.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

Problem 1 (Irrational logarithm) (20 points).
Prove that $\log_{50} 20$ is irrational. You may use whatever familiar facts about integers and primes you need (Unique Factorization, for example), but please explicitly state such facts, and explain how they are used in your proof.

Problem 2 (Truth Tables) (20 points).

Claim. *There are exactly two truth environments (assignments) for the variables M, N, P, Q, R, S that satisfy the following formula:*

$$\underbrace{(\overline{P} \text{ OR } Q)}_{\textit{clause (1)}} \text{ AND } \underbrace{(\overline{Q} \text{ OR } R)}_{\textit{clause (2)}} \text{ AND } \underbrace{(\overline{R} \text{ OR } S)}_{\textit{clause (3)}} \text{ AND } \underbrace{(\overline{S} \text{ OR } P)}_{\textit{clause (4)}} \text{ AND} M \text{ AND } \overline{N}\,.$$

(a) This claim could be proved by truth-table. How many rows would the truth table have? _______

(b) Instead of a truth-table, prove this claim with an argument by cases according to the truth value of P.

Problem 3 (Normal Forms) (20 points).
The five-variable propositional formula

$$P ::= (A \text{ AND } B \text{ AND } \overline{C} \text{ AND } D \text{ AND } \overline{E}) \text{ OR } (\overline{A} \text{ AND } B \text{ AND } \overline{C} \text{ AND } \overline{E})$$

is in Disjunctive Normal Form with two "AND-of-literal" clauses.

(a) Find a **Full** Disjunctive Normal Form that is equivalent to P, and explain your reasoning.

Hint: Can you narrow in on the important parts of the truth table without writing all of it? Alternatively, can you avoid the truth table altogether?

(b) Let C be a **Full Con**junctive Normal Form that is equivalent to P. Assume that C has been simplified so that none of its "OR-of-literals" clauses are equivalent to each other. How many clauses are there in C? (Please **don't** try to write out any of these clauses.)

Briefly explain your answer.

Problem 4 (Domains of Discourse) (20 points).
Consider these five domains of discourse, in order:

$\mathbb{N}$ (nonnegative integers), $\mathbb{Z}$ (integers), $\mathbb{Q}$ (rationals), $\mathbb{R}$ (reals), $\mathbb{C}$ (complex numbers).

For each of the logic formulas below, indicate the first of these domains where the formula is true, or state "**none**" if it is not true in any of them. Please briefly explain each one.

i. $\forall x \exists y.\, y = 3x$ _______

ii. $\forall x \exists y.\, 3y = x$ _______

iii. $\forall x \exists y.\, y^2 = x$ _______

iv. $\forall x \exists y.\, y < x$ _______

v. $\forall x \exists y.\, y^3 = x$ _______

vi. $\forall x \neq 0.\, \exists y, z.\, y \neq z \text{ AND } y^2 = x = z^2$ _______

Problem 5 (Predicate Formulas) (20 points).
Some (but not necessarily all) students from a large class will be lined up left to right. There will be at least two students in the line. Translate each of the following assertions into predicate formulas, using quantifiers $\exists$ and $\forall$ and any logical operators like AND, OR, NOT, etc., with the set of students in the class as the domain of discourse. The only predicates you may use about students are

- equality, and
- $F(x, y)$, meaning that "x is somewhere to the left of y in the line." For example, in the line "cda", both $F(c, a)$ and $F(c, d)$ are true. (A student is not considered to be "to the left of" themself, so $F(x, x)$ is always false.)

As a worked example, the predicate "there are at least two students after x" may be written as

$$\exists y\, \exists z.\ \text{NOT}(y = z) \text{ AND } (F(x, y) \text{ AND } F(x, z)).$$

In your answers you may use earlier predicates in later parts, even if you did not solve the earlier parts. So your answer to part (c) might refer to inline(x) and/or first(x), for example.

(a) Student x is in the line. Call this predicate inline(x).

Hint: Since there are at least 2 students in the line, inline(x) means x is to the left or right of someone else.

(b) Student x is first in line. Call this predicate first(x).

(c) Student x is immediately to the right of student y. Call this predicate isnext(x, y).

Hint: No other student can be between x and y.

(d) Student x is second in line.