

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Integral Method for Sums



Albert R Meyer, April 10, 2013

integralsum.1

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Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

n^{th} Harmonic number

$$B_n = H_n/2$$

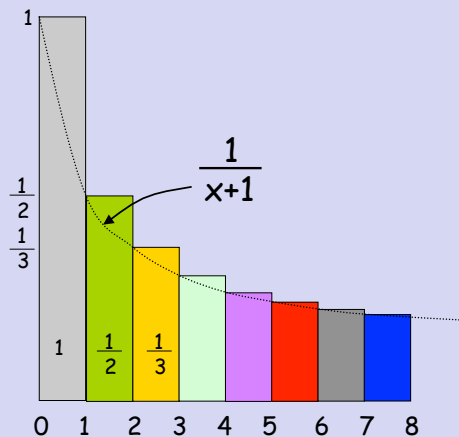


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integralsum.2

6	9	13	7
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Integral estimate for H_n



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integralsum.3

6	9	13	7
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Integral estimate for H_n

$H_n =$ area of rectangles
> area under $1/(x+1) =$

$$\int_0^n \frac{1}{x+1} dx = \int_1^{n+1} \frac{1}{x} dx = \ln(n+1)$$



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integralsum.4

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Book stacking

for overhang 3, need $B_n \geq 3$

$$H_n \geq 6$$

integral bound: $\ln(n+1) \geq 6$

so ok with $n \geq \lceil e^6 - 1 \rceil = 403$ books

actually calculate H_n :

227 books are enough.



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April 10, 2013

integralsum.7

6	9	13	7
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Book stacking

$\log(n+1) \rightarrow \infty$ as $n \rightarrow \infty$,

so overhang can be

as big as desired!



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April 10, 2013

integralsum.8

6	9	13	7
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CD cases over the edge



43 cases high --top 4 cases completely off the table --1.8 or 1.9 case-lengths



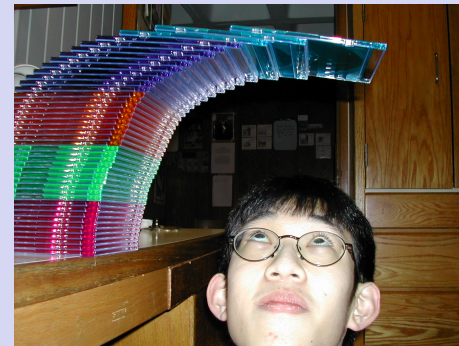
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integralsum.9

6	9	13	7
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stack of 43 CD's



see 6.042 Spring02 demo page for credits



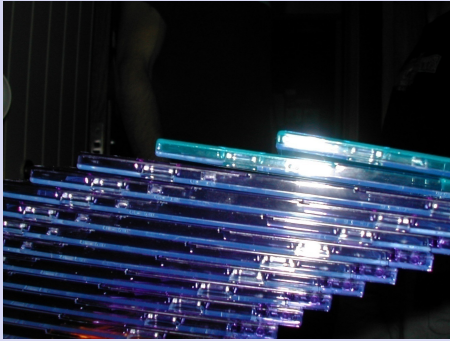
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integralsum.10

6	9	13	7
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don't sneeze

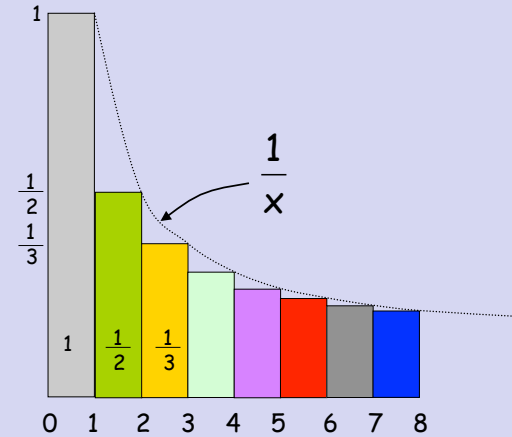


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integralsum.11

6	9	13	7
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Upper bound for H_n



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integralsum.12

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Integral Sum Bounds

Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a weakly decreasing function.

$$S ::= \sum_{i=1}^n f(i), \quad I ::= \int_1^n f(x) dx$$

$$I + f(n) \leq S \leq I + f(1)$$



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integralsum.13

6	9	13	7
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Upper bound for H_n

$$H_n < \left(\int_1^n \frac{1}{x} dx \right) + 1$$

$$= 1 + \ln(n)$$



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integralsum.14

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Asymptotic bound for H_n

$$\ln(n+1) < H_n < 1 + \ln(n)$$

$$H_n \sim \ln(n)$$



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integralsum.15

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Asymptotic Equivalence

Def: $f(n) \sim g(n)$

$$\lim_{n \rightarrow \infty} \left(\frac{f(n)}{g(n)} \right) = 1$$



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integralsum.16

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Asymptotic Equivalence \sim

Example: $(n^2 + n) \sim n^2$

pf:

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right) = 1$$



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April 10, 2013

integralsum.17