Mathematics for Computer Science
MIT 6.042J/18.062J

Integral Method for Sums

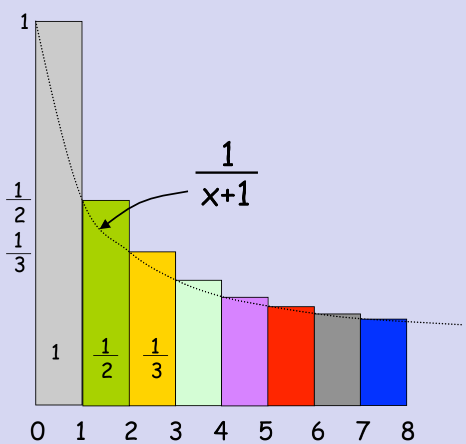
Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

n^{th} Harmonic number

$$B_n = H_n/2$$

Integral estimate for H_n



Integral estimate for H_n

H_n = area of rectangles

> area under 1/(x+1) =

$$\int_0^n \frac{1}{x+1}\,dx = \int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$$

Book stacking

for overhang 3, need $B_n \geq 3$

$H_n \geq 6$

integral bound: $\ln(n+1) \geq 6$

so ok with $n \geq \lceil e^6 - 1 \rceil = 403$ books

actually calculate H_n:

227 books are enough.

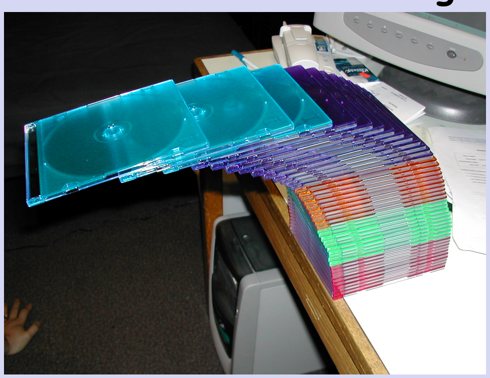
Book stacking

$\log(n+1) \to \infty$ as $n \to \infty$,

so overhang can be

as big as desired!

CD cases over the edge

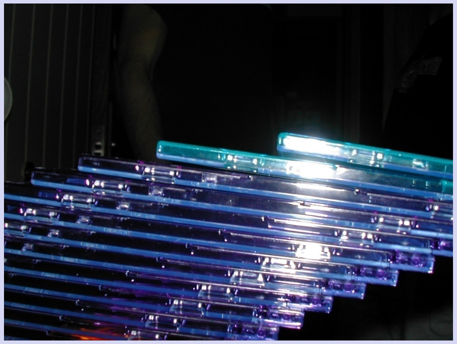


43 cases high --top 4 cases completely off the table --1.8 or 1.9 case-lengths

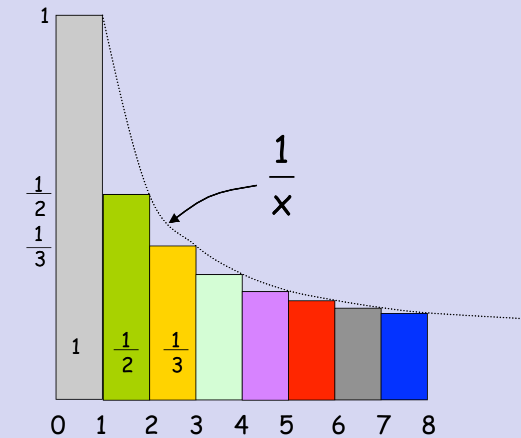
stack of 43 CD's

see 6.042 Spring02 demo page for credits

don't sneeze



Upper bound for H_n



Integral Sum Bounds

Let $f:\mathbb{R}^+ \to \mathbb{R}^+$ be a weakly decreasing function.

$$S ::= \sum_{i=1}^{n} f(i), \quad I ::= \int_1^n f(x)dx$$

$$I + f(n) \le S \le I + f(1)$$

Upper bound for H_n

$$H_n < \left(\int_1^n \frac{1}{x} dx\right) + 1$$

$$= 1 + \ln(n)$$

Asymptotic bound for H_n

$$\ln(n+1) < H_n < 1 + \ln(n)$$

$$H_n \sim \ln(n)$$

Asymptotic Equivalence

Def: $f(n) \sim g(n)$

$$\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = 1$$

Asymptotic Equivalence ~

Example: $(n^2 + n) \sim n^2$

pf:

$$\lim_{n \to \infty} \frac{n^2 + n}{n^2} = \lim_{n \to \infty} \left(1 + \frac{1}{n} \right) = 1$$