Mathematics for Computer Science
MIT 6.042J/18.062J

Generalized Counting Rules

Generalized Product Rule

\# lineups of 5 students in class
let S::= students
say |S| = 91 so
~~|lineups of 5 students|~~ NO!
student can't be in 2 places:
|seqs in S^5 with no repeats| ?

Generalized Product Rule

|seqs in S^5 with no repeats|
91 choices for 1st student,
90 choices for 2nd student,
89 choices for 3rd student,
88 choices for 4th student,
87 choices for 5th student

$$= 91 \cdot 90 \cdot 89 \cdot 88 \cdot 87 = \frac{91!}{86!}$$

Generalized Product Rule

Q a set of length-k sequences
if n_1 possible 1^{st} elements,
n_2 possible 2^{nd} elements
(for each first entry),
n_3 possible 3^{rd} elements
(for each 1^{st} & 2^{nd} entry,...)
then, $|Q| = n_1 \cdot n_2 \cdots n_k$

Division Rule

#6.042 students =

$$\frac{\text{\#6.042 students' fingers}}{10}$$

Division Rule

if function from A to B

is k-to-1, then

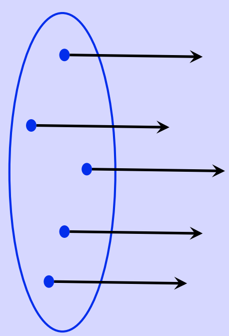
$$|A| = k|B|$$

(generalized Bijection Rule)

Division Rule

function from A to B

A



arrows

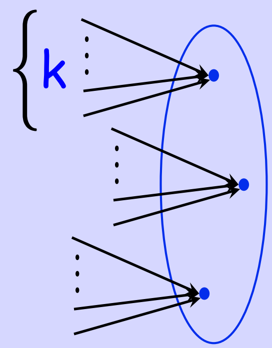
$= |A|$

Division Rule

function is k-to-1

arrows

$= k|B|$



B

Counting Subsets

How many size 4 subsets of {1,2,…,13}?

{1,2,3,4}
{1,2,3,5}
{3,4,7,11}
⋮
?

Counting Subsets

How many size 4 subsets of {1,2,…,13}?

Let A::= permutations of {1,2,…,13}

B::= size 4 subsets

map $a_1\, a_2\, a_3\, a_4\, a_5 \ldots a_{12}\, a_{13} \in A$

to $\{a_1, a_2, a_3, a_4\} \in B$

Counting Subsets

$a_1\, a_3\, a_2\, a_4\, a_5 \ldots a_{12}\, a_{13}$ also maps

to $\{a_1, a_2, a_3, a_4\}$

so does $\underbrace{a_1\, a_3\, a_2\, a_4}_{4!\text{ perms}}\, \underbrace{a_{13} \ldots a_{12}\, a_5}_{9!\text{ perms}}$

all map to same set

$4! \cdot 9!$-to-1

Counting Subsets

$$13! = |A| = (4! \cdot 9!)|B|$$

so # of size 4 subsets is

$$\binom{13}{4} ::= \frac{13!}{4!9!}$$

Counting Subsets

m element subsets of an n element set is

$$\binom{n}{m} ::= \frac{n!}{m!(n-m)!}$$

n choose m