Mathematics for Computer Science
MIT 6.042J/18.062J

Proof by Cases: Friends & Strangers

Friends & Strangers

Six people. Every two are either friends or strangers.

Claim: there is a set of

3 mutual friends or

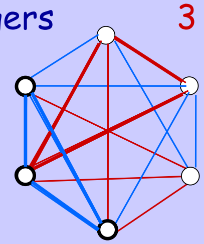
3 mutual strangers

Friends & Strangers

People are circles

3 mutual strangers

3 mutual friends



red line shows friends
blue line shows strangers

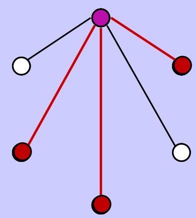
Friends & Strangers

Take 3 minutes to find a counter-example

--or convince yourself there isn't any counterexample, that is, the Claim is true.

A Proof of the Claim

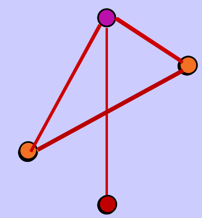
- Person ● has a line to each of the 5 other people.
- lines are red or blue, so at least 3 must be the same color.



● has ≥ 3 friends

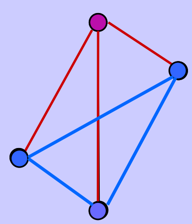
A Proof of the Claim

Case 1: some pair of these friends are friends of each other, then we have 3 mutual friends:



A Proof of the Claim

Case 2: no pair of these friends are friends of each other, so we have 3 mutual strangers:



A Proof of the Claim

Since the Claim is true in either case, and one of these cases always holds, the Claim is always true.

QED

Ramsey's Theorem

For any k, every *large enough* group of people will include either

k mutual friends, or
k mutual strangers.

Ramsey's Theorem

For any k, every *large enough* group of people will include either

size-k red *clique*, or
size-k blue *clique*.

Let R(k) be the *large enough* size.

So we've proved that $R(3) \leq 6$.

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Let R(k) be the *large enough* size.

In fact, $R(3) = 6$.

(Just show group of 5 w/o Δ, Δ)

Ramsey's Numbers

Turns out that R(4) = 18 (not easy!)
R(5) is unknown!
Paul Erdös considered finding R(6) a hopeless challenge!

So in our second class, we have reached a research frontier!