Final Examination, Unit 1

Your name: _____

- This exam is **closed book** except for two 2-sided cribsheets. Total time is 180 minutes.

- Write your solutions in the space provided. If you need more space, **write on the back** of the sheet containing the problem.

- In answering the following questions, you may use without proof any of the results from class or text.

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	20		
2	15		
3	15		
Total	50		

Problem 1 (**Proof by Cases**) (**20 points**).

Define the function

$$f(x) ::= 2\,|x + 2| - |x - 3| - |x + 4|$$

for real numbers x. Carefully prove that

$$-7 \leq f(x) \leq 3 \quad \text{for all } x \in \mathbb{R},$$

using a **proof by cases** based on the value of x.

   Hint: $|x + 4|$ equals $x + 4$ when $x \geq -4$ and equals $-(x + 4)$ when $x \leq -4$. You should have 4 cases.

Problem 2 (**Truth Tables and Normal Forms**) (**15 points**). **(a)** Draw a truth table for the formula

$$A \text{ IMPLIES NOT}(B \text{ IMPLIES } C).$$

Be sure to show your work.

 (b) Use part (a) to write a **Full Disjunctive Normal Form** that is equivalent to the formula given above. No explanation is required.

Problem 3 (**Logical Formulas**) (**15 points**).
Let $Q(x, y)$ be the statement

"*x* has been a contestant on television show *y*."

The domain of discourse for *x* is the set of all students at your school and for *y* is the set of all quiz shows that have ever been on television.

Indicate which of the following expressions are logically equivalent to the sentence:

"No student at your school has ever been a contestant on a television quiz show."

(a) $\forall x \, \forall y. \; \text{NOT}(Q(x, y))$

(b) $\exists x \, \exists y. \; \text{NOT}(Q(x, y))$

(c) $\text{NOT}(\forall x \, \forall y. \; Q(x, y))$

(d) $\text{NOT}(\exists x \, \exists y. \; Q(x, y))$

(e) Write your own logical formula that is equivalent to the statement "No student at your school has been a contestant on *two or more* television quiz shows." You may use variables y_1, y_2 that both range over the domain of television quiz shows.