Mathematics for Computer Science
MIT 6.042J/18.062J

Equivalence Relations

Albert R Meyer March 22, 2013 equiv.1

two-way walks

walk from u to v and back from v to u:
u and v are strongly connected.

$u\ G^*\ v$ AND $v\ G^*\ u$

Albert R Meyer March 22, 2013 equiv.2

symmetry

relation R on set A is symmetric iff

$a\ R\ b$ IMPLIES $b\ R\ a$

Albert R Meyer March 22, 2013 equiv.3

equivalence relations

transitive,
symmetric &
reflexive

Albert R Meyer March 22, 2013 equiv.4

equivalence relations

Theorem:

R is an equiv rel iff
R is the strongly connected relation of some digraph

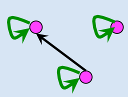
equivalence relations

examples:

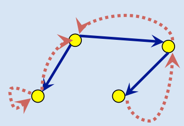
- $=$ (equality)
- $\equiv$ (mod n)
- same size
- same color

Graphical Properties of Relations

Reflexive

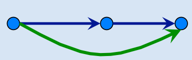


Asymmetric

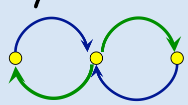


NO

Transitive



Symmetric



Representing Equivalences

Representing equivalences

For total function $f:A \to B$

define relation $\equiv_f$ on A:

$a \equiv_f a'$ IFF $f(a) = f(a')$

Representing equivalences

Theorem:

Relation R on set A is an equiv. relation IFF

R is $\equiv_f$

for some $f:A \to B$

representing $\equiv \pmod{n}$

$\equiv \pmod{n}$ is

$\equiv_f$ where

$f(k) ::= \text{rem}(k,n)$

Representing equivalences

For partition Π of A

define relation $\equiv_\Pi$ on A:

$a \equiv_\Pi a'$ IFF a, a' are in the same block of Π

Representing equivalences

Theorem:

Relation R on set A is an equiv. relation IFF

$$R \text{ is } \equiv_{\Pi}$$

for some partition Π of A