

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

Digital Logic



Albert R Meyer

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digital.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Adding in binary

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 1000011
 \end{array}$$

1 1 1 ← carry



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February 13, 2015

digital.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Adding in binary

$$\begin{array}{r}
 39 \text{ is } 100111 \\
 28 \text{ is } 011100 \\
 \hline
 \text{sum} = 67 \text{ is } 1000011
 \end{array}$$



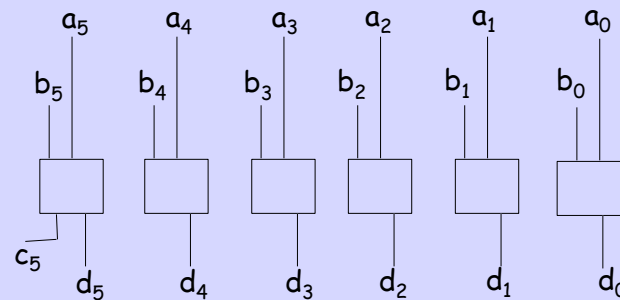
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digital.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary addition circuit



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digital.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary addition circuit

b_5 1 b_4 0 b_3 0 b_2 1 b_1 1 b_0 1
 c_5 d_5 c_4 d_4 c_3 d_3 c_2 d_2 c_1 d_1 c_0 d_0

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary addition circuit

a_5 a_4 a_3 a_2 a_1 a_0
 b_5 b_4 b_3 b_2 b_1 b_0
 c_5 d_5 c_4 d_4 c_3 d_3 c_2 d_2 c_1 d_1 c_0 d_0

"ripple carry"

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary addition circuit

a_5 a_4 a_3 a_2 a_1 a_0
 b_5 b_4 b_3 b_2 b_1 b_0
 c_5 d_5 c_4 d_4 c_3 d_3 c_2 d_2 c_1 d_1 c_0 d_0

"ripple carry"

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

half Adder

$d ::= a \text{ XOR } b$
 $c ::= a \text{ AND } b$

from [http://en.wikipedia.org/wiki/Adder_\(electronics\)](http://en.wikipedia.org/wiki/Adder_(electronics))
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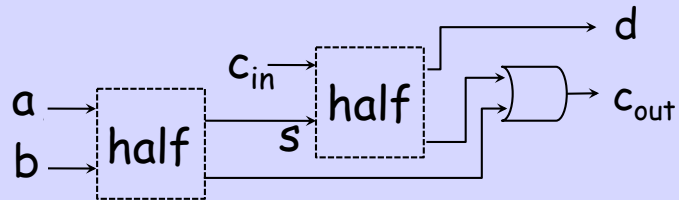
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

full Adder

$$s ::= a \text{ XOR } b$$

$$d ::= c_{in} \text{ XOR } s$$

$$c_{out} ::= (c_{in} \text{ AND } s) \text{ OR } (a \text{ AND } b)$$



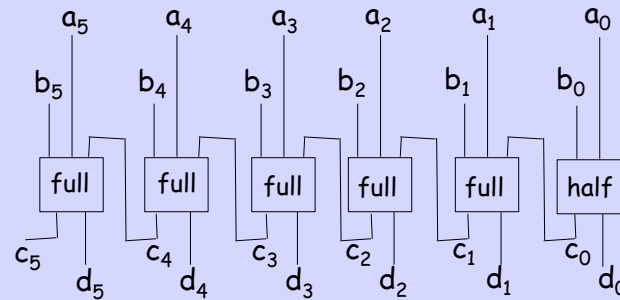
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digital.11

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary addition circuit



"ripple carry"



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digital.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Ripple Carry formulas

$$d_0 ::= a_0 \text{ XOR } b_0$$

$$c_0 ::= a_0 \text{ AND } b_0$$

$$s_i ::= a_i \text{ XOR } b_i$$

$$d_i ::= c_{i-1} \text{ XOR } s_i$$

$$c_i ::= (c_{i-1} \text{ AND } s_i) \text{ OR } (a_i \text{ AND } b_i)$$



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digital.13