Mathematics for Computer Science
MIT 6.042J/18.062J

Bounds on Deviation
Markov Bound

Example: IQ

IQ measure was constructed so that

average IQ = 100.

What fraction of the people can possibly have an IQ ≥ 300?

...at most 1/3

IQ Higher than 300?

If more than 1/3 have $IQ \geq 300$, then

$avg > (1/3) \cdot 300 = 100$!

—a contradiction

IQ Higher than 300?

At most 1/3 of people have $IQ \geq 300$

$$Pr[IQ \geq 300] \leq \frac{E[IQ]}{300}$$

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IQ Higher than x?

In general,

$$Pr[IQ \geq x] \leq \frac{100}{x}$$

IQ Higher than x?

Besides mean = 100,
we used only one fact about
the distribution of IQ:

IQ is always nonnegative

Markov Bound

If R is nonnegative, then

$$Pr[R \geq x] \leq \frac{E[R]}{x}$$

for $x > E[R]$

Markov Bound (Restated)

Let $x = c \cdot E[R]$:

$$Pr[R \geq c\mu] \leq \frac{1}{c}$$

$$Pr[R \geq 3 \cdot expected]$$
$$\leq 1/3$$

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Markov Bound

- Weak
- Obvious
- Useful anyway

IQ ≥ 300, again

Suppose we are given that
IQ is always ≥ 50?
Get a better bound using
(IQ – 50)
since this is now ≥ 0.

IQ ≥ 300, again

$$\Pr[IQ \geq 300] =$$
$$\Pr[IQ - 50 \geq 300 - 50]$$
$$\leq \frac{100 - 50}{300 - 50} = \frac{1}{5}$$

Improved Markov Bound

Better bound from
Markov by shifting R
to have 0 as minimum