

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Bounds on Deviation Markov Bound



Albert R Meyer,

May 10, 2013

markov.1

6	9	13	7
12	10	5	
3	1	4	14
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Example: IQ

IQ measure was constructed so that

average IQ = 100.

What fraction of the people can possibly have an IQ ≥ 300 ?

...at most $1/3$



Albert R Meyer,

May 10, 2013

markov.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

IQ Higher than 300?

If more than $1/3$ have
IQ ≥ 300 , then
avg $> (1/3) \cdot 300 = 100$!
—a contradiction



Albert R Meyer,

May 10, 2013

markov.3

6	9	13	7
12	10	5	
3	1	4	14
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IQ Higher than 300?

At most $1/3$ of people
have IQ ≥ 300

$$\Pr[\text{IQ} \geq 300] \leq \frac{E[\text{IQ}]}{300}$$



Albert R Meyer,

May 10, 2013

markov.5

6	9	13	7
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IQ Higher than x ?

In general,

$$\Pr[\text{IQ} \geq x] \leq \frac{100}{x}$$



Albert R Meyer,

May 10, 2013

markov.6

6	9	13	7
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IQ Higher than x ?

Besides mean = 100,
we used only one fact about
the distribution of IQ:
IQ is always nonnegative



Albert R Meyer,

May 10, 2013

markov.7

6	9	13	7
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Markov Bound

If R is nonnegative, then

$$\Pr[R \geq x] \leq \frac{E[R]}{x}$$

for $x > E[R]$



Albert R Meyer,

May 10, 2013

markov.8

6	9	13	7
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Markov Bound (Restated)

Let $x = c \cdot E[R]$:

$$\Pr[R \geq c\mu] \leq \frac{1}{c}$$

$$\Pr[R \geq 3 \cdot \text{expected}] \leq 1/3$$



Albert R Meyer,

May 10, 2013

markov.9

6	9	13	7
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Markov Bound

- Weak
- Obvious
- Useful anyway



Albert R Meyer,

May 10, 2013

markov.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

IQ ≥ 300 , again

Suppose we are **given** that IQ is always ≥ 50 ?

Get a better bound using
(IQ - 50)
since this is now ≥ 0 .



Albert R Meyer,

May 10, 2013

markov.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

IQ ≥ 300 , again

$$\begin{aligned} \Pr[\text{IQ} \geq 300] &= \\ \Pr[\text{IQ} - 50 \geq 300 - 50] & \\ \leq \frac{100 - 50}{300 - 50} &= \frac{1}{5} \end{aligned}$$



Albert R Meyer,

May 10, 2013

markov.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Improved Markov Bound

Better bound from Markov by shifting **R** to have **0** as minimum



Albert R Meyer,

May 10, 2013

markov.13