

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science

MIT 6.042J/18.062J

Countable Sets



Albert R Meyer, March 4, 2015

countable.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Countable Sets

A is countable iff can be listed a_0, a_1, a_2, \dots

same as \mathbb{N} bij A or A finite

so \mathbb{Z}^+ , \mathbb{Z} countable



Albert R Meyer, March 4, 2015

countable.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binary words are countable

$\{0,1\}^*$::= finite binary words

list the (empty) string of length 0

list the 2 length-1 bit strings

then list the 2^2 length-2 bit strings
(in binary notation order)

then the 2^3 length-3 bit strings

⋮



Albert R Meyer, March 4, 2015

countable.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

$\mathbb{N} \times \mathbb{N}$ is countable

start with $(0,0)$

then $(0,1), (1,0)$

then $(0,2), (2,0), (1,1)$

then $(0,3), (3,0), (1,2), (2,1)$

⋮

then all pairs with sum n



Albert R Meyer, March 4, 2015

countable.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Proving Countability

Lemma: A is countable iff
can list A allowing repeats:

$$\mathbb{N} \text{ surj } A$$

Corollary: A is countable iff

$$C \text{ surj } A$$

for some countable C



Albert R Meyer, March 4, 2015

countable.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Rationals are countable

map (m,n) to $\frac{m}{n}$

$$\underbrace{\mathbb{N} \times \mathbb{N}} \text{ surj } \underbrace{\mathbb{Q}^{\geq 0}}$$

countable so countable



Albert R Meyer, March 4, 2015

countable.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Reals are uncountable

But $\{0,1\}^\omega$ and the

real numbers \mathbb{R}

are **not** countable:

next lecture.



Albert R Meyer, March 4, 2015

countable.7