Mathematics for Computer Science
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Countable Sets

Countable Sets

A is countable iff can be listed $a_0, a_1, a_2, \ldots$

same as $\mathbb{N}$ bij A or A finite

so $\mathbb{Z}^+$, $\mathbb{Z}$ countable

Binary words are countable

$\{0,1\}^*$ ::= finite binary words

list the (empty) string of length 0

list the 2 length-1 bit strings

then list the 2^2 length-2 bit strings (in binary notation order)

then the 2^3 length-3 bit strings

$\vdots$

$\mathbb{N} \times \mathbb{N}$ is countable

start with $(0,0)$

then $(0,1)$, $(1,0)$

then $(0,2)$, $(2,0)$, $(1,1)$

then $(0,3)$, $(3,0)$, $(1,2)$, $(2,1)$

$\vdots$

then all pairs with sum n

Proving Countability

Lemma: A is countable iff can list A allowing repeats:

$$\mathbb{N} \text{ surj } A$$

Corollary: A is countable iff

$$C \text{ surj } A$$

for some countable C

Rationals are countable

map (m,n) to $\dfrac{m}{n}$

$$\underbrace{\mathbb{N} \times \mathbb{N}}_{\text{countable}} \text{ surj } \underbrace{\mathbb{Q}^{\geq 0}}_{\text{countable}}$$

countable so countable

Reals are uncountable

But $\{0,1\}^{\omega}$ and the real numbers $\mathbb{R}$ are not countable: next lecture.