

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mathematics for Computer Science
MIT 6.042J/18.062J

Bookkeeper Rule Multinomial Theorem



Albert R Meyer, April 22, 2013

bookkeeper.1

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

bookkeeper rule

permutations of the word
bookkeeper ?

- # perms $bo_1o_2k_1k_2e_1e_2e_3r = 10!$
- map perm $o_1be_1o_2k_1rk_2e_2e_3$ to $obeokrkepe$
- 2 o's, 2 k's, 3 e's:
map is $2! \cdot 2! \cdot 3!$ -to-1 $\frac{10!}{2!2!3!}$



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bookkeeper.2

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

bookkeeper rule

permutations of length- n
word with n_1 a's, n_2 b's, ..., n_k z's:

$$\binom{n}{n_1, n_2, \dots, n_k} ::= \frac{n!}{n_1! n_2! \dots n_k!}$$

multinomial coefficient



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bookkeeper.3

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 15 | 8 | 11 | 2 |

binomial coefficients

binomial a special case:

$$\binom{n}{k} = \binom{n}{n-k}$$



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bookkeeper.4

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

multinomials

What is the coefficient of
 EMS^3TY
 in the expansion of
 $(E + M + S + T + Y)^7$?

The number of ways to
 rearrange the letters in
 the word
 $SYSTEMS$



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bookkeeper.6

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

applying the BOOKKEEPER rule

What is the coefficient of
 EMS^3TY
 in the expansion of
 $(E + M + S + T + Y)^7$?

$$\binom{7}{1, 1, 3, 1, 1}$$



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bookkeeper.7

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

multinomial coefficients

What is the coefficient of
 BA^3N^2
 in the expansion of
 $(B + A + N)^6$?

The number of ways to
 rearrange the letters in
 the word
 $BANANA$



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bookkeeper.10

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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multinomial coefficients

What is the coefficient of
 BA^3N^2
 in the expansion of
 $(B + A + N)^6$?

$$\binom{6}{1, 3, 2}$$



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bookkeeper.11

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 15 | 8 | 11 | 2 |

multinomial coefficients

Take 14 mile walk including 3 Northward miles, 4 Southward, 5 Eastward and 3 Westward. How many different walks?

= #rearrangements of

$$N^3 S^4 E^5 W^2$$

$$= \binom{14}{3, 4, 5, 2}$$



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bookkeeper.12

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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multinomial coefficients

What is the coefficient of

$$X_1^{r_1} X_2^{r_2} X_3^{r_3} \dots X_k^{r_k}$$

in the expansion of

$$(X_1 + X_2 + X_3 + \dots + X_k)^n ?$$

$$\binom{n}{r_1, r_2, r_3, \dots, r_k}$$



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bookkeeper.13

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 15 | 8 | 11 | 2 |

The Multinomial Formula

$$(X_1 + X_2 + \dots + X_k)^n =$$

$$\sum_{r_1 + \dots + r_k = n} \binom{n}{r_1, r_2, r_3, \dots, r_k} X_1^{r_1} X_2^{r_2} X_3^{r_3} \dots X_k^{r_k}$$



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bookkeeper.14

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

multinomial coefficients

$$\binom{n}{r_1, r_2, r_3, \dots, r_k}$$

$$::= 0 \quad \text{if } r_1 + r_2 + \dots + r_k \neq n$$



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bookkeeper.15

| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
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Preceding slides adapted from:

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bookkeeper.17