

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science  
MIT 6.042J/18.062J

# Book Stacking Harmonic Sums

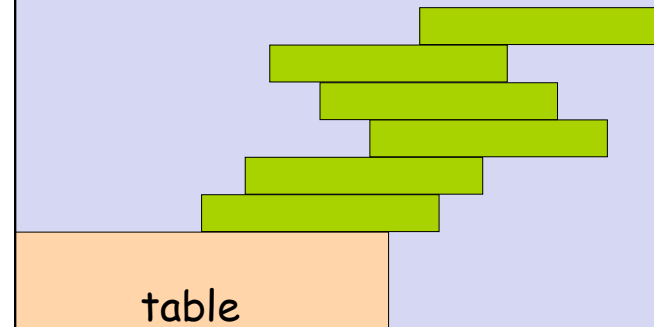


Albert R Meyer, April 6, 2012

lec 8F.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Book Stacking



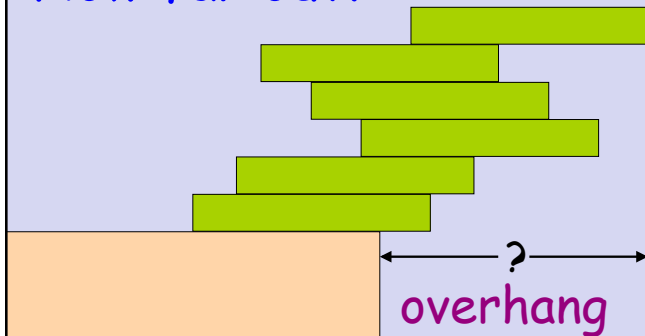
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Book Stacking

How far out?



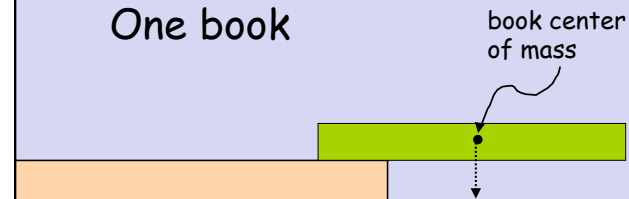
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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

## Book Stacking

One book



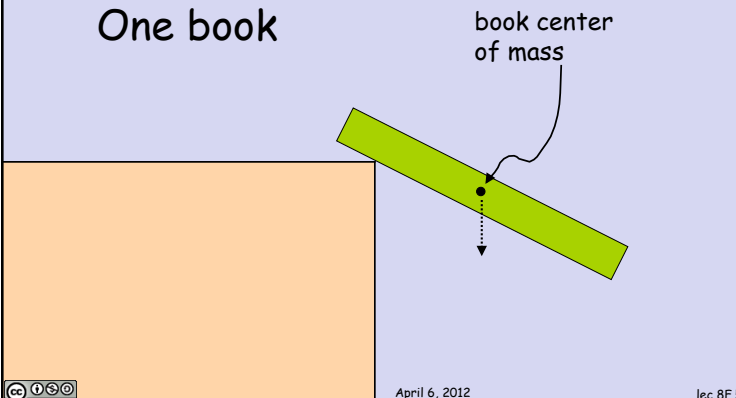
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lec 8F.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book Stacking

One book



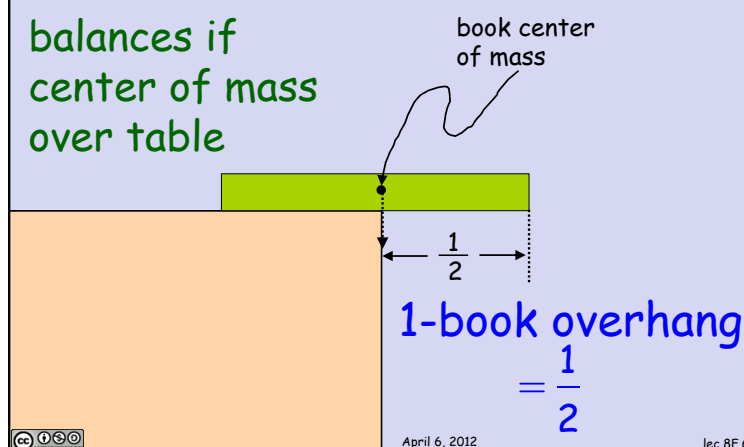
book center  
of mass

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book Stacking

balances if  
center of mass  
over table



book center  
of mass

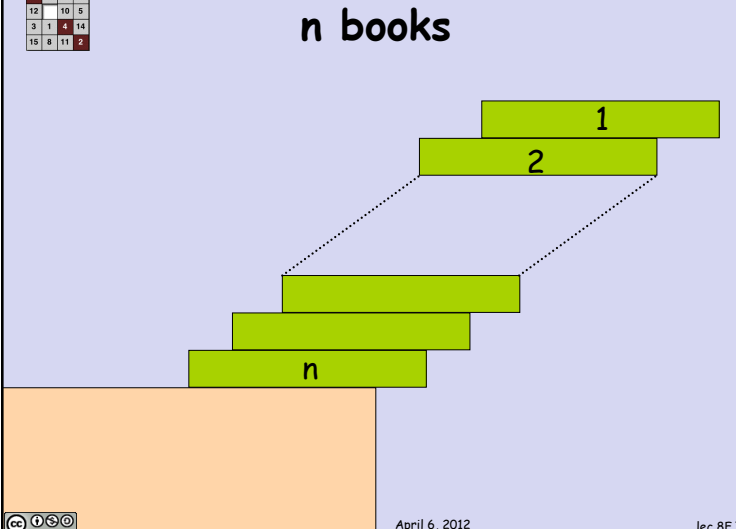
$\frac{1}{2}$

1-book overhang  
 $= \frac{1}{2}$

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## n books

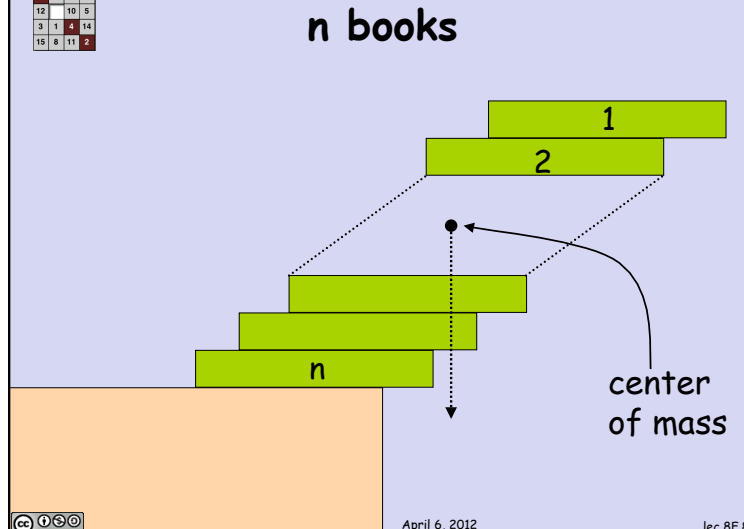


1  
2  
n

April 6, 2012 lec 8F.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## n books



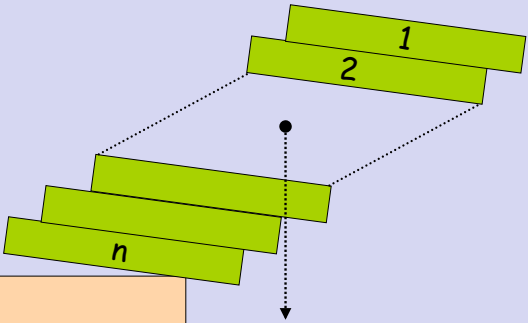
1  
2  
n

center  
of mass

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

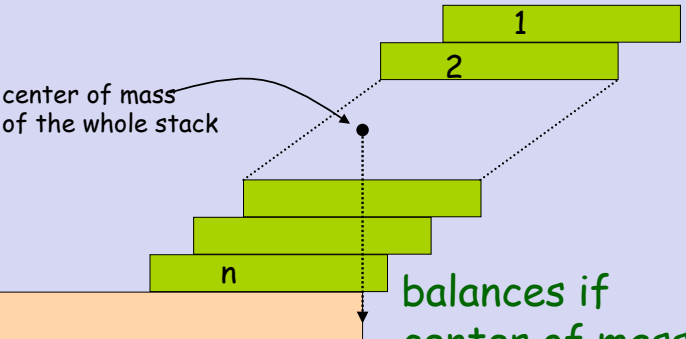
## n books



April 6, 2012 lec 8F.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

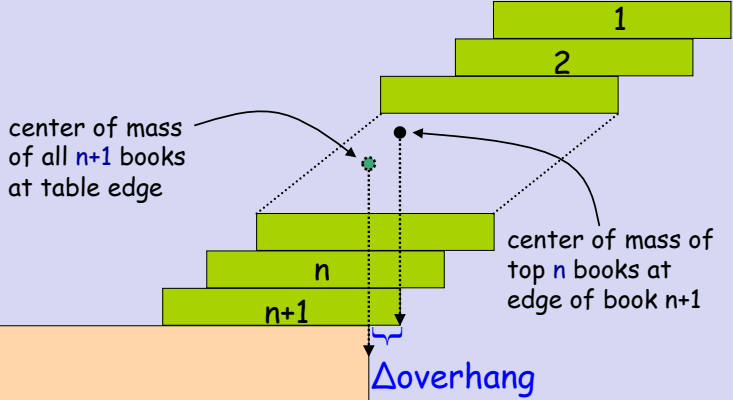
## n books



April 6, 2012 lec 8F.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## n+1 books



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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

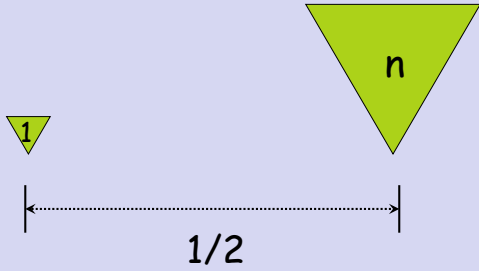
## $\Delta$ -overhang

$\Delta$ -overhang ::= horizontal distance from n-book to (n+1)-book centers of mass

Albert R Meyer, April 6, 2012 lec 8F.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Δ-overhang



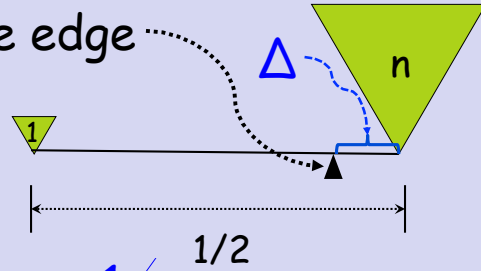
$1/2$

Albert R Meyer, April 6, 2012 lec 8F.15

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Δ-overhang

table edge



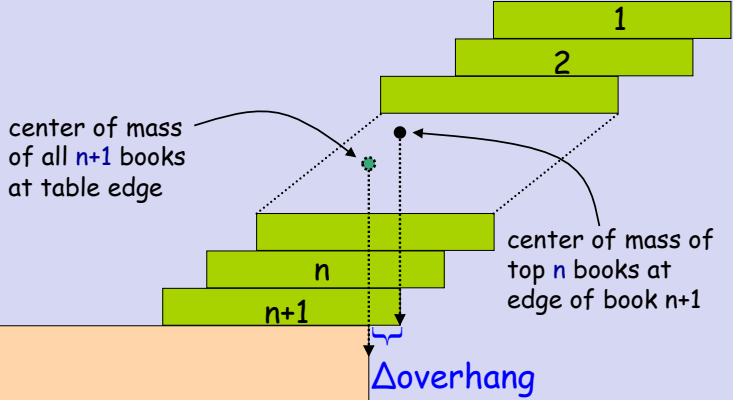
$1/2$

$$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$$

Albert R Meyer, April 6, 2012 lec 8F.16

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## n+1 books



center of mass of all  $n+1$  books at table edge

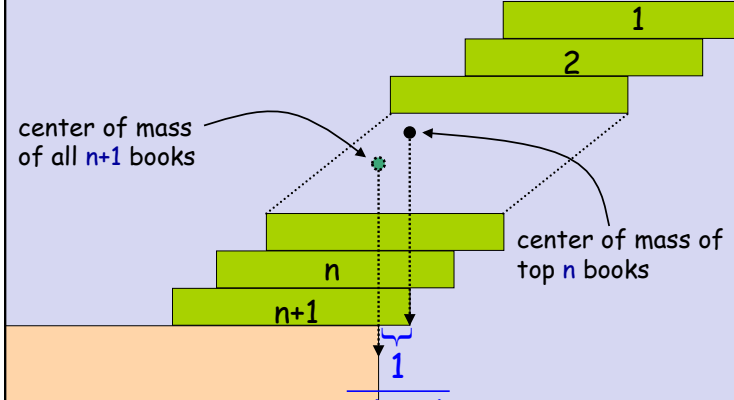
center of mass of top  $n$  books at edge of book  $n+1$

$\Delta$ overhang

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## n+1 books



center of mass of all  $n+1$  books

center of mass of top  $n$  books

$\frac{1}{2(n+1)}$

April 6, 2012 lec 8F.18

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book stacking summary

$B_n$  ::= overhang of  $n$  books

$$B_1 = 1/2$$

$$B_{n+1} = B_n + \frac{1}{2(n+1)}$$

$$B_n = \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$



Albert R Meyer,

April 6, 2012

lec 8F.19

6	5	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$n^{\text{th}}$  Harmonic number

$$B_n = H_n/2$$



Albert R Meyer,

April 6, 2012

lec 8F.20