

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Bipartite Matching

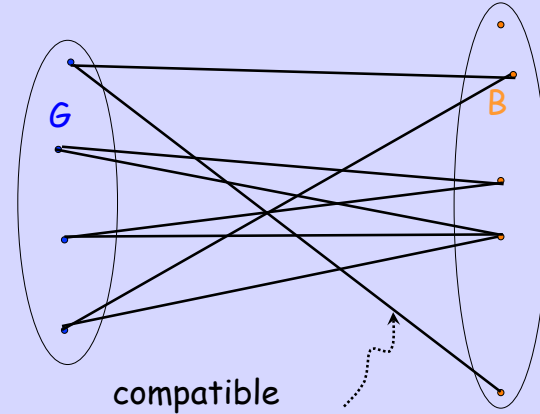


Albert R Meyer, April 3, 2013

bipartite.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

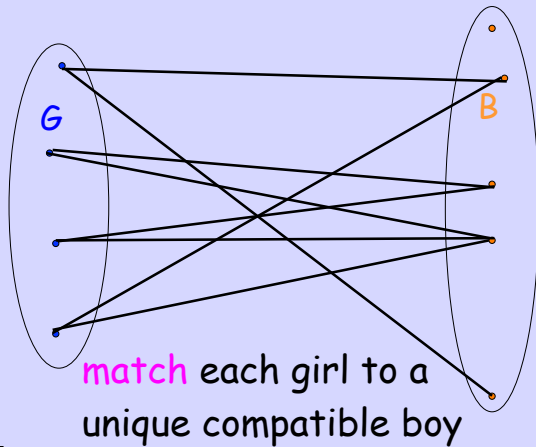


Albert R Meyer, April 3, 2013

bipartite.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

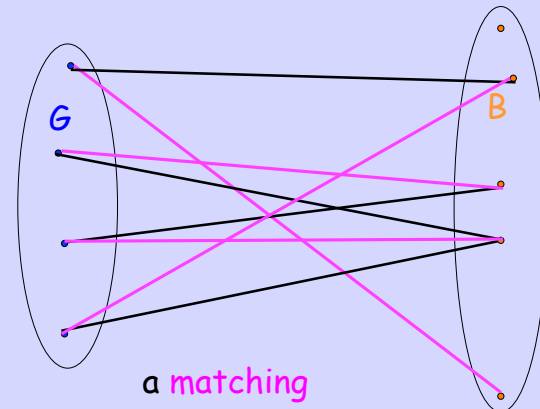


Albert R Meyer, April 3, 2013

bipartite.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls



Albert R Meyer, April 3, 2013

bipartite.4

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

suppose this **edge** was missing

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

suppose this **edge** was missing

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Compatible Boys & Girls

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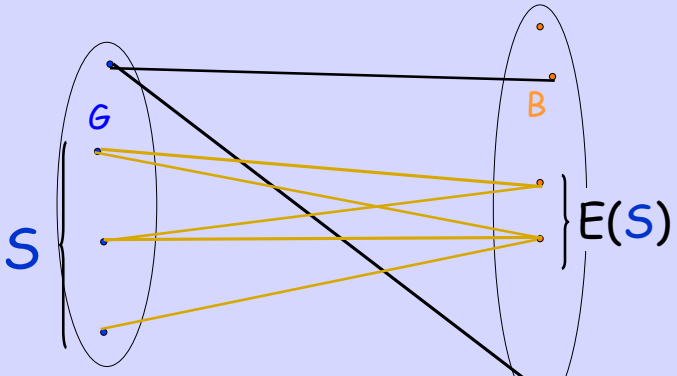
6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Not enough boys for these girls!

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

No match is possible!



$|S| = 3 > 2 = |E(S)|$

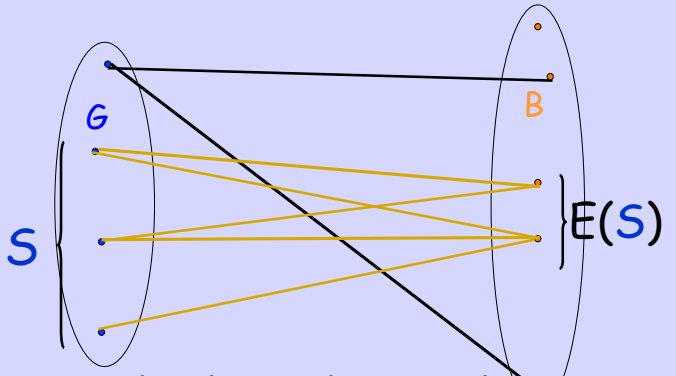
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bipartite.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

a bottleneck



$|S| > |E(S)|$

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bipartite.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Bottleneck Lemma

Bottleneck: a set S of girls without enough boys.

$E(S) ::=$ boys adjacent to at least one girl in S .

$|S| > |E(S)|$

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lec 7M.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Bottleneck Lemma

If there **is** a **bottleneck**, then **no match** is possible **obviously**

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bipartite.12

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Hall's Theorem

Conversely, if there are
no bottlenecks, then
there is a **match**.
Not obvious. Discuss next.

