Mathematics for Computer Science
MIT 6.042J/18.062J

Bijections for Counting

Example: Counting Passwords

Password conditions:

- characters are digits & letters
- between 6 & 8 characters long
- starts with a letter
- case sensitive

Counting Passwords

$L ::= \{a,b,\ldots,z,A,B,\ldots,Z\}$

$D ::= \{0,1,\ldots\ldots,9\}$

P_n ::= length n words starting w/letter

$= L \times (L \cup D)^{n-1}$

Counting Passwords

$|L \times (L \cup D)^{n-1}|$

$= |L| \cdot |(L \cup D)|^{n-1}$

$= |L| \cdot (|L| + |D|)^{n-1}$

$= 52 \cdot (52+10)^{n-1}$

Counting Passwords

set of passwords:

$$P ::= P_6 \cup P_7 \cup P_8$$

$$|P| = |P_6|+|P_7|+|P_8|$$

$$= 52\cdot(62^5+62^6+62^7)$$

$$\approx 19\cdot 10^{14}$$

4-digit nums w/ $\geq$ one 7

cases by 1st occurrence of 7:

x: any digit   o: any digit $\neq$ 7

7xxx or o7xx or oo7x or ooo7

$$10^3 + 9\cdot 10^2 + 9^2\cdot 10 + 9^3$$

$$= 3439$$

at least one 7: another way

|4-digit nums w/ $\geq$ one 7|

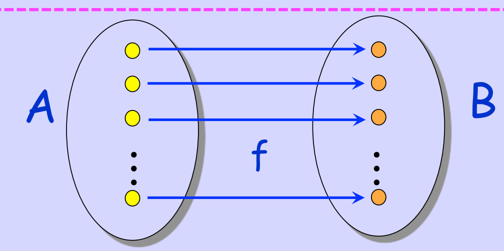
= |4-digit nums|

— |those w/ no 7|

$$= 10^4 - 9^4 = 3439$$

Mapping Rule: Bijections

If f is a bijection from A to B, then $|A| = |B|$



Bijection: $\mathcal{P}(A)$ and Binary Strings

A: $\{a_1, a_2, a_3, a_4, a_5, \ldots, a_n\}$

subset: $\{a_1, \quad a_3, a_4, \quad \ldots, a_n\}$

string: 1 0 1 1 0 … 1

This is a bijection, so

$\underbrace{|n\text{-bit binary strings}|}_{2^n} = |\mathcal{P}(A)|$

Size of P(A)

$$|\mathcal{P}(A)| = 2^{|A|}$$

Counting Doughnut Selections

From 5 kinds of doughnuts select a dozen.

let A ::= all selections of 12 doughnuts

00	(none)	000000	00	00
chocolate	lemon	sugar	glazed	plain

Counting Doughnut Selections

B::= 16-bit words with four 1's

0011000000100100

00	1		1	000000	1	00	1	00
chocolate		lemon		sugar		glazed		plain

Bijection from A to B

c chocolate, l lemon, s sugar, g glazed, p plain

maps to

$$0^c 1 0^l 1 0^s 1 0^g 1 0^p$$

so $|A| = |B|$

Albert R Meyer, April 17, 2013