

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Asymptotic Properties



Albert R Meyer,

April 10, 2013

Oh-props.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The Oh's

lemma:

If $f = o(g)$ or $f \sim g$, then $f = O(g)$

$\lim = 0$ OR $\lim = 1$ IMPLIES $\lim < \infty$



Albert R Meyer,

April 10, 2013

Oh-props.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

The Oh's

If $f = o(g)$, then $g \neq O(f)$

$\lim \frac{f}{g} = 0$ IMPLIES $\lim \frac{g}{f} = \infty$



Albert R Meyer,

April 10, 2013

Oh-props.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

Lemma: $x^a = o(x^b)$ for $a < b$

Proof: $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$ and $b - a > 0$

so as $x \rightarrow \infty$, $\frac{1}{x^{b-a}} \rightarrow 0$



Albert R Meyer,

April 10, 2013

Oh-props.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

Lemma:

$$\ln x = o(x^\varepsilon)$$

for $\varepsilon > 0$.



Albert R Meyer,

April 10, 2013

Oh-props.6

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

Lemma: $\ln x = o(x^\varepsilon)$ for $\varepsilon > 0$.

Proof:

$$\frac{1}{y} \leq y \text{ for } y \geq 1 \text{ so } \int_1^z \frac{1}{y} dy \leq \int_1^z y dy$$

$$\ln z \leq \frac{z^2}{2} \text{ for } z \geq 1$$



Albert R Meyer,

April 10, 2013

Oh-props.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

Lemma: $\ln x = o(x^\varepsilon)$ for $\varepsilon > 0$.

Proof: $\ln z \leq \frac{z^2}{2}$, so let $z = \sqrt{x^\delta}$

$$\frac{\delta \ln x}{2} \leq \frac{x^\delta}{2}$$

$$\ln x \leq \frac{x^\delta}{\delta} = o(x^\varepsilon) \text{ for } \varepsilon > \delta.$$



Albert R Meyer,

April 10, 2013

Oh-props.8

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

Lemma:

$$x^c = o(a^x)$$

for $a > 1$.



Albert R Meyer,

April 10, 2013

Oh-props.9

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Little Oh: $o(\cdot)$

proofs:
L'Hopital's Rule,
McLaurin Series
(see a Calculus text)



Albert R Meyer,

April 10, 2013

Oh-props.10

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Big Oh: $O(\cdot)$

Equivalent definition:

$$f(n) = O(g(n))$$

$$\exists c, n_0 \forall n \geq n_0.$$

$$f(n) \leq c \cdot g(n)$$



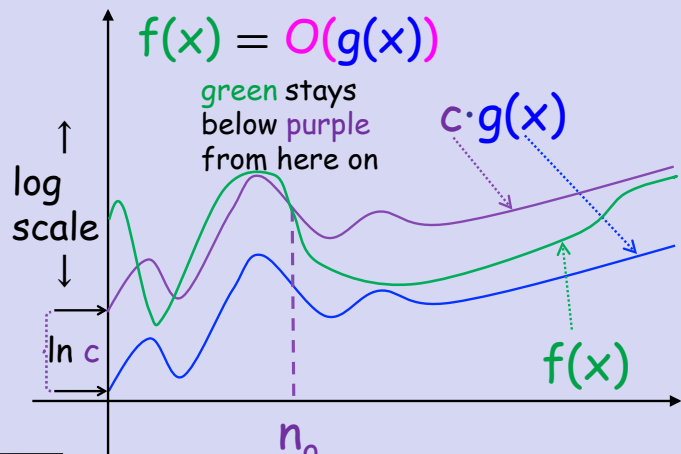
Albert R Meyer,

April 10, 2013

Oh-props.11

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Big Oh: $O(\cdot)$



Albert R Meyer,

April 10, 2013

Oh-props.12

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Why limsup?

If $f \leq 2g$ then $f = O(g)$,
but maybe f/g has no limit.

example $f(n) = \left(1 + \sin^2\left(\frac{n\pi}{2}\right)\right) \cdot g(n)$

but

$$\limsup_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 2 < \infty$$



Albert R Meyer,

April 10, 2013

Oh-props.13