

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Arithmetic Sums



Albert R Meyer, April 10, 2013

arithmetic-sum.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sum for Children

$$\begin{array}{r}
 89 + 102 + 115 + 128 + 141 + \\
 154 + \quad \quad \quad \quad \quad \quad + \\
 193 + \quad \quad \quad \quad \quad \quad + \\
 232 + \quad \quad \quad \quad \quad \quad + \\
 323 + \quad \quad \quad \quad \quad \quad + \\
 414 + \quad \quad \quad \quad \quad \quad \dots + 453 + 466
 \end{array}$$



Albert R Meyer, April 10, 2013

arithmetic-sum.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

C. F. Gauss



Picture source: <http://www-groups.dcs.st-and.ac.uk/~history/PictDisplay/Gauss.html>



Albert R Meyer, April 10, 2013

arithmetic-sum.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Sum for Children

Nine-year old Gauss saw
30 numbers, each 13 greater
than the previous one.
(So the story goes.)



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arithmetic-sum.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum for Children

$$89 + (89+13) + \dots + (89+29 \cdot 13)$$

$$F + (F+d) + \dots + (L-d) + L = A$$

$$L + (L-d) + \dots + (F+d) + F = A$$

$$(F+L) + (F+L) + \dots + (F+L) + (F+L) = 2A$$

$$A = \frac{(F+L)}{2} \cdot (\# \text{ terms})$$

average term



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April 10, 2013

arithmetic-sum.7

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Sum for Children

Example:

$$1 + 2 + \dots + (n-1) + n =$$

$$\frac{n(1+n)}{2}$$



Albert R Meyer,

April 10, 2013

arithmetic-sum.10