

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

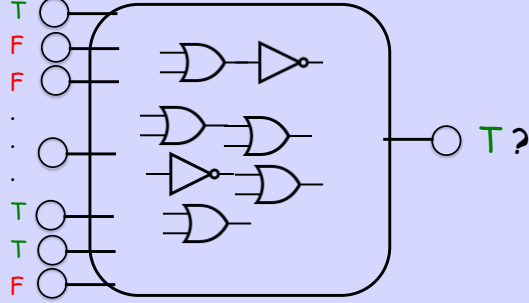
Mathematics for Computer Science
 MIT 6.042J/18.062J

SAT Reduces to 3-Coloring

Albert R Meyer, October 23, 2017

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT



Is there an assignment of T's and F's to the inputs that yields output T?

Albert R Meyer, October 23, 2017

SATcolor.2

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT

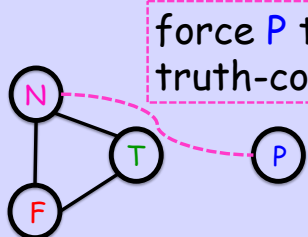
Create graph whose 3-colorings simulate circuit behavior

Albert R Meyer, October 23, 2017

SATcolor.3

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

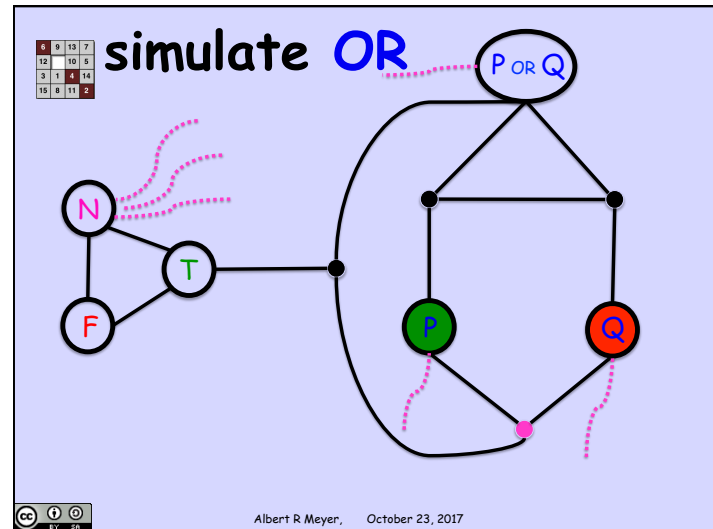
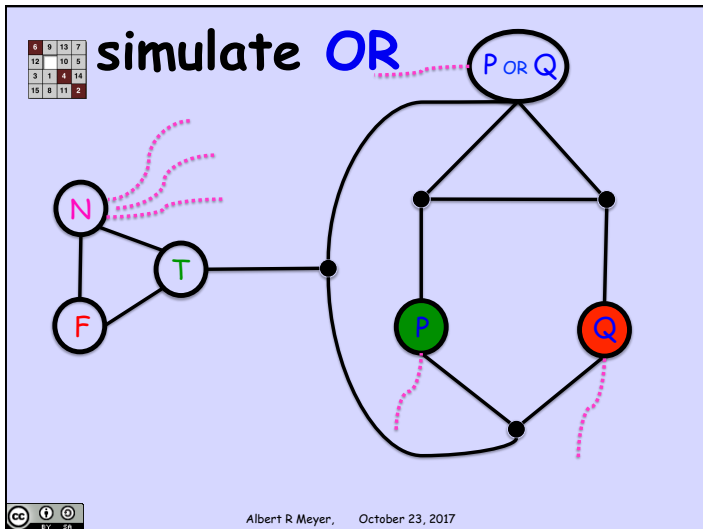
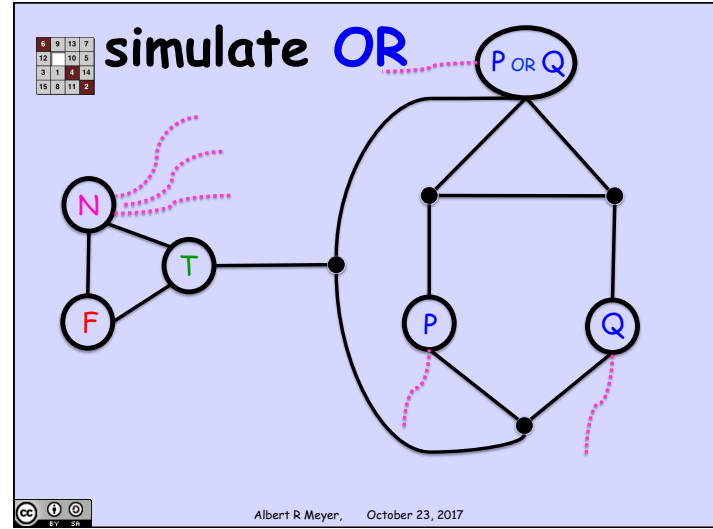
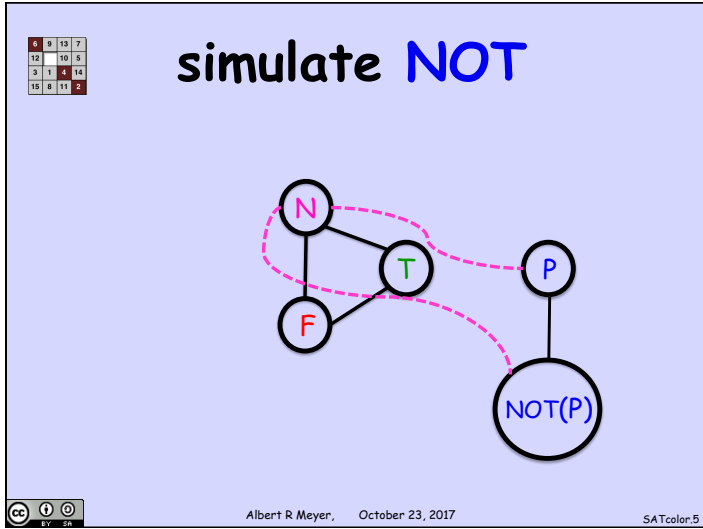
Truth Colors

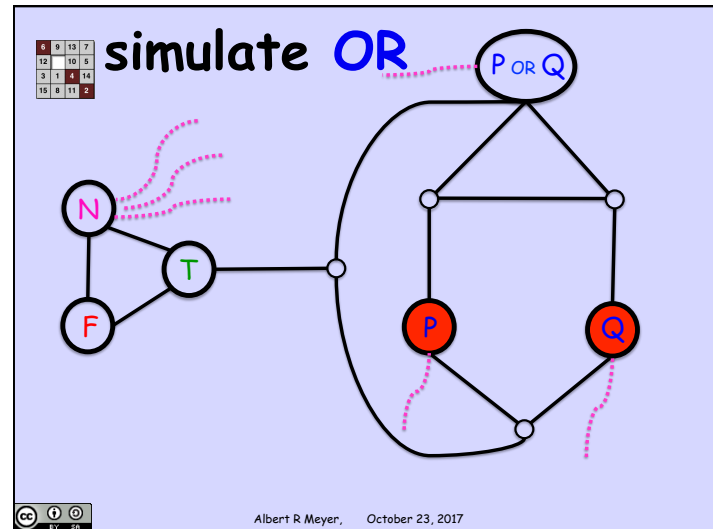
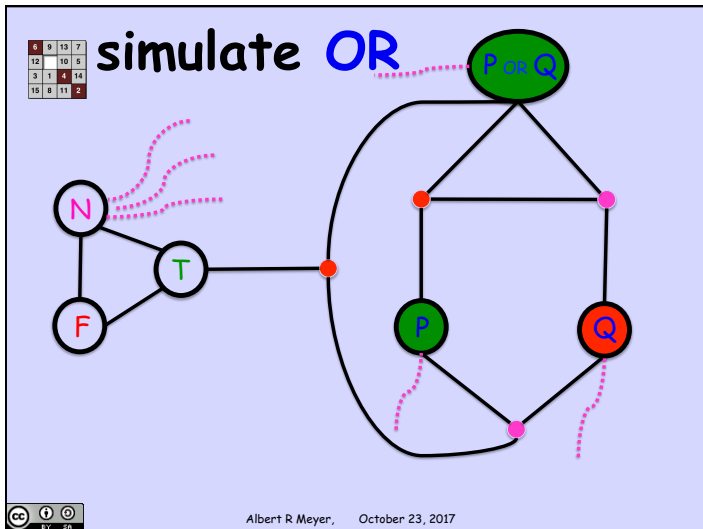
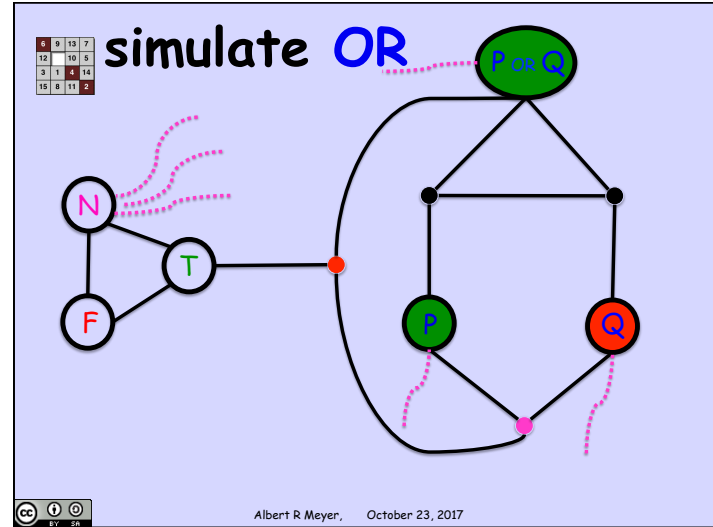
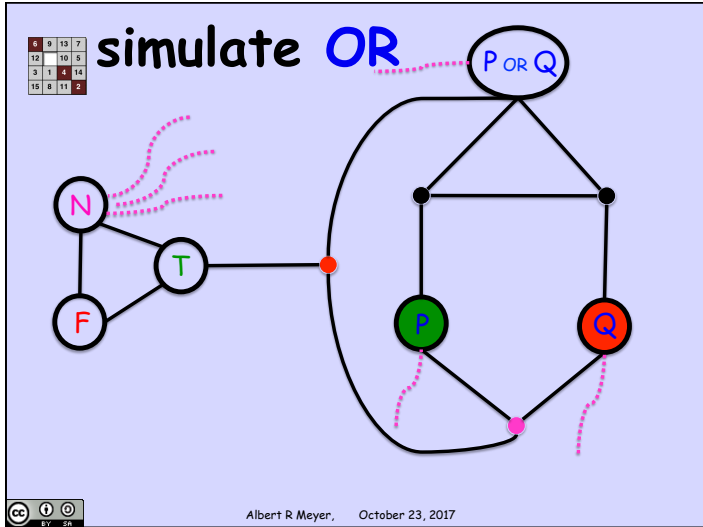


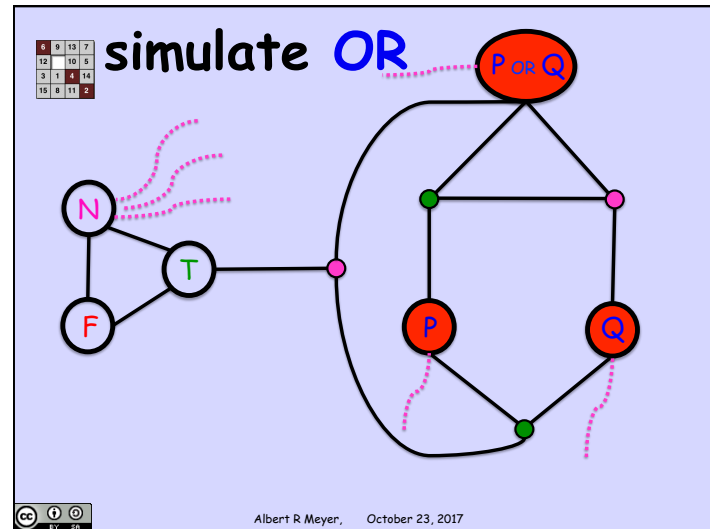
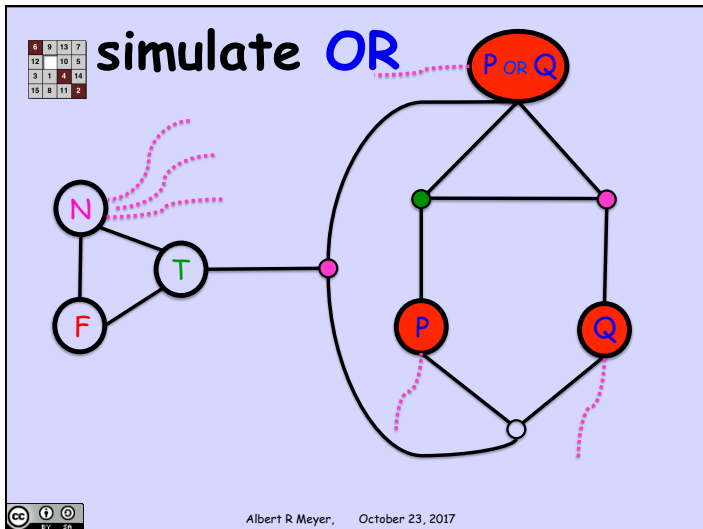
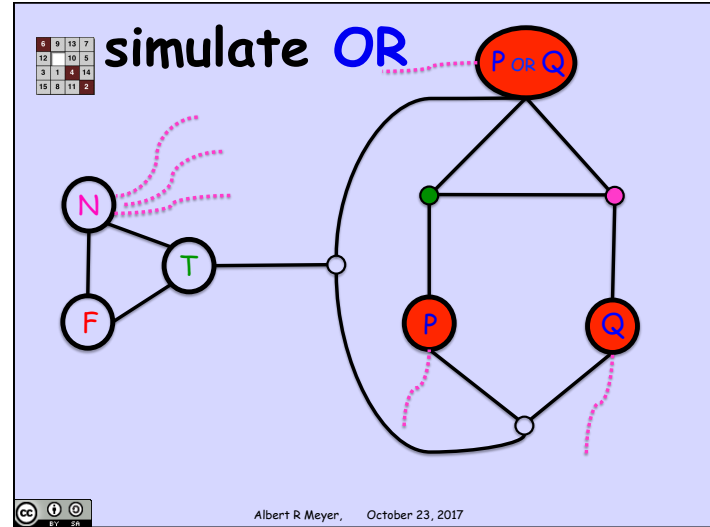
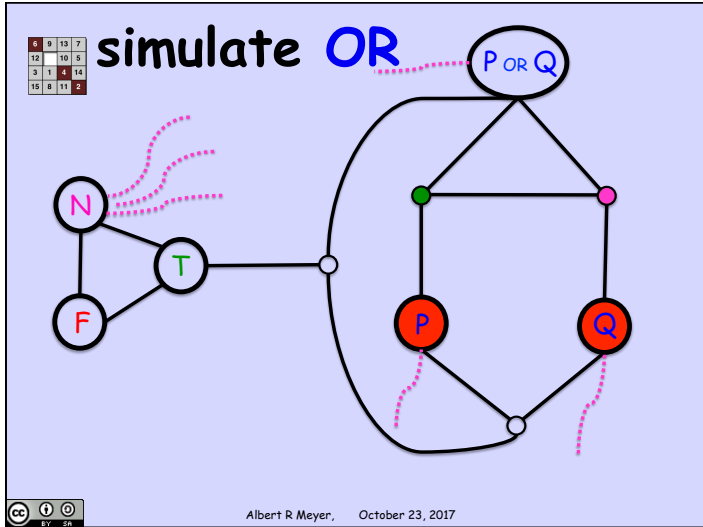
force P to be truth-colored

Albert R Meyer, October 23, 2017

SATcolor.4

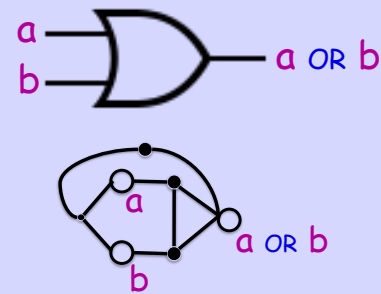






| | | | |
|----|----|----|----|
| 6 | 9 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

OR gate & gadget

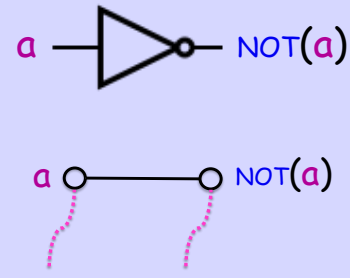


Albert R Meyer, October 23, 2017

SATcolor:18

| | | | |
|----|----|----|----|
| 6 | 9 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

NOT gate & gadget

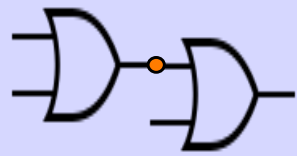


Albert R Meyer, October 23, 2017

SATcolor:19

| | | | |
|----|----|----|----|
| 6 | 9 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT

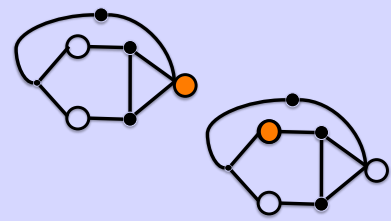


Albert R Meyer, October 23, 2017

SATcolor:20

| | | | |
|----|----|----|----|
| 6 | 9 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT

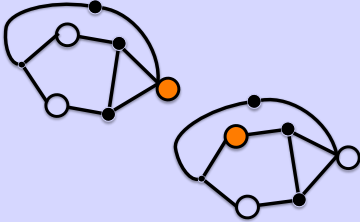


Albert R Meyer, October 23, 2017

SATcolor:21

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT



CC BY SA

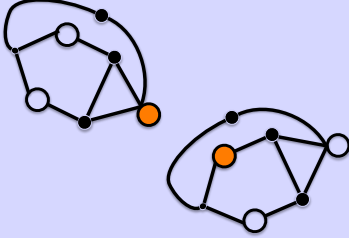
Albert R Meyer, October 23, 2017

SATcolor.22

The diagram shows two identical circuit components connected in series. Each component consists of a network of nodes and edges. The left component has a single orange node on its right side. The right component has a single orange node on its left side, which connects to the right component's orange node.

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT



CC BY SA

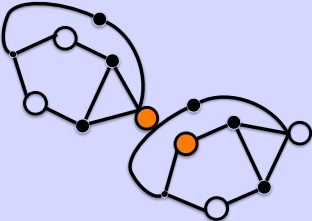
Albert R Meyer, October 23, 2017

SATcolor.23

The diagram shows two identical circuit components connected in series. The left component has a single orange node on its right side. The right component has two orange nodes on its left side, which connect to the left component's orange node.

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT



CC BY SA

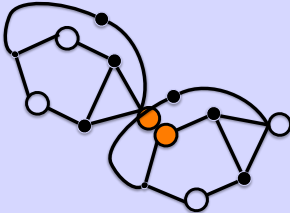
Albert R Meyer, October 23, 2017

SATcolor.24

The diagram shows two identical circuit components connected in series. The left component has a single orange node on its right side. The right component has two orange nodes on its left side, which connect to the left component's orange node. Additionally, the right component has a single orange node on its right side.

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Circuit SAT



CC BY SA

Albert R Meyer, October 23, 2017

SATcolor.25

The diagram shows two identical circuit components connected in series. The left component has a single orange node on its right side. The right component has two orange nodes on its left side, which connect to the left component's orange node. Additionally, the right component has two orange nodes on its right side.

Circuit SAT

Albert R Meyer, October 23, 2017 SATcolor.26

Circuit SAT

replace gates by gadgets

Albert R Meyer, October 23, 2017 SATcolor.27

Circuit SAT

replace gates by gadgets

Albert R Meyer, October 23, 2017 SATcolor.28

Circuit SAT

force output T


graph is 3-colorable iff circuit is in SAT

Albert R Meyer, October 23, 2017 SATcolor.29

| | | | |
|----|----|----|----|
| 6 | 8 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 11 | 11 | 2 |

SAT vs 3-Color

SAT reduces to **3-Color**:
any fast procedure for
3-Color would yield a fast
SAT procedure.




Albert R Meyer, October 23, 2017

SATcolor.31

| | | | |
|----|----|----|----|
| 6 | 8 | 12 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 11 | 11 | 2 |

SAT vs 3-Color

Theorists -- and world
institutions -- are betting there
isn't any fast **SAT** procedure,
so we can safely bet there
isn't any fast **3-Coloring**
procedure either.



Albert R Meyer, October 23, 2017

SATcolor.33